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Synthesis of Adaptive Sliding Mode Control for Twin Rotor MIMO System with Mass Uncertainty based on Synergetic Control Theory

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ABSTRACT

In this paper, the authors present a new method to synthesize an adaptive sliding controller for Twin Rotor MIMO System (TRMS) based on Synergetic Control Theory (SCT). This system represents a prototype of a helicopter with two degrees of freedom and is widely used in automatic control laboratories. The complexity of the control problem is due to the nonlinear cross-coupling between the main and tail rotors. Uncertainty in system parameters further increases the complexity of the control problem. In Synergetic Control Theory, manifolds are designed for each channel. The control law is found based on sequential manifolds and the Analytical Design of Aggregated Regulators (ADAR) method. The adaptive law when the parameters are uncertain is given based on the analysis of system stability thanks to the Lyapunov function of the first manifold. Finally, the effectiveness of the proposed controller is demonstrated by numerical simulation results and comparison with conventional Sliding Mode Control (SMC).

1. Introduction

Twin Rotor MIMO System is a prototype model of a helicopter with two degrees of freedom in the laboratory. The TRMS diagram is depicted in Fig. 1, which includes a horizontal bar that rotates around a fixed distance. The system is driven by two perpendicular propellers located at its two ends and driven by a DC motor. The pitch angle of the propeller is fixed, so the thrust is governed by the rotation speed of the propeller. TRMS is characterized by highly nonlinear cross-coupled dynamics and disturbances due to the motion of air currents, leading to the complexity of control problems.

Helicopter system control has been studied extensively using many different control techniques. In research [1], [2] using conventional PID controller and PD controller combined with genetic optimization algorithm. In research [3], [4], [40], [41] a linear quadratic regulator (LQR) is used for this system and research [5] presents a model predictive controller (MPC). These control laws are designed based on linear models or working point proximity models, so the control quality still has certain limitations. In studies [6]-[8], [42]-[48] control laws based on nonlinear control methods were presented, which gave quite impressive results. The sliding mode controller is presented in the studies [6]-[8], the results show the high stability of the control law, low system response time, but the sliding mode control law always has charactering phenomenon even though it has been applied. There are techniques to overcome this phenomenon. The backstepping control law presented in studies [8], [9] ensures a stable system with high sustainability. But this control law has a large response time. Controllers based on fuzzy theory and neural networks are presented in works [10], [11], the results show the effectiveness of these controllers when the system has model and model uncertainty impact noise. But it is difficult to improve control quality. In practice, one often has to consider cases where the helicopter system is uncertain, as the mass may change during operation. As is known, adaptive control is a useful and important approach to deal with system uncertainties due to its ability to provide on-line estimates of unknown system parameters using measurement. Adaptive control of helicopter systems has been studied in [12], [15]. In [12], model reference adaptive control (MRAC) is designed for a linear helicopter system. However, considering the linearized system makes the controller design simpler and reduces the control quality when far from the equilibrium point. In research [13], [14], the overall adaptive backstepping controller method presented quite good results, but the response time was still quite large. Sliding mode control and its variants are widely used in many studies [15], [31]-[39]. Adaptive sliding mode control presented in studies [15] shows impressive results.

Since the synergetic control theory was proposed by Professor A.A. Kolecnikov, it has been widely used to design controllers for many different objects [16]-[30]. This theory has a number of advantages over conventional methods, such as synthesis steps based on a quasi-natural process, with each input signal driving the system to a specified manifold. In this theory, the desired values are considered immutable. The main method of SCT control system design is the ADAR method. While using this method, the control law design ensures the movement of the invariant manifold of the closed-loop system from an arbitrary initial state to the vicinity of the desired invariant manifold (the attractive points). Thereby, the control law not only implements the necessary invariance but also ensures the asymptotic stability of the entire system. This shows that the control law is effective even when the model has an uncertainty component, but it does not completely eliminate this effect. With those advantages, the design of controllers and adaptive control have been published in several studies [20]-[30].

In this paper, a mathematical model of TRMS is found from the Euler-Lagrange equations with pitch and yaw axes, including the uncertainty parameter. The adaptive sliding mode control based on the synergetic control theory is designed. The stability of the proposed adaptive control law is proved using the first manifold-based Lyapunov functions. The performance of the proposed method is compared with the conventional SMC. The results show that the control criteria respond better to the SMC controller. The simulation results show the effectiveness, stability, and controllability of the proposed control law.

2. Synthesis of Adaptive Sliding Control based on Synergetic Control Theory for a TRMS

2.1. Mathematical Model of a TRMS

This TRMS consists of a rod rotated on its base such that it can rotate freely in both the horizontal and vertical planes (Fig. 1). There are two rotors (main rotor and tail rotor), driven by DC motors, at each end of the rod. Two rotors driven by variable-speed electric motors allow the TRMS to rotate in the vertical and horizontal planes (pitch and yaw angles). The potential energy (P) due to gravity and the total kinetic energy (T) due to moment of inertia, are given by

$$\begin{aligned} &P = mgl_0 sin\theta; \quad T = T_{r\_p} + T_{r\_y} + T_t; \quad T_{r\_p} = \frac{1}{2} J_{eq\_p}\dot{\theta}^2 \\ &T_t = \frac{1}{2} m(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2); \quad T_{r\_y} = \frac{1}{2} J_{eq\_y}\dot{\psi}^2 \end{aligned} \tag{1}$$

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Where Tr_p and Tr_y are the sum of the rotational kinetic energies acting from the pitch and yaw respectively, and Tt is the translational kinetic energy produced by the center of mass. Jeq_p and Jeq_y are the moments of inertia of the pitch and yaw motor respectively. m is the total moving mass of TRMS, g is gravitational acceleration. l0 is the distance from the center of gravity of the TRMS to the rotation axis.

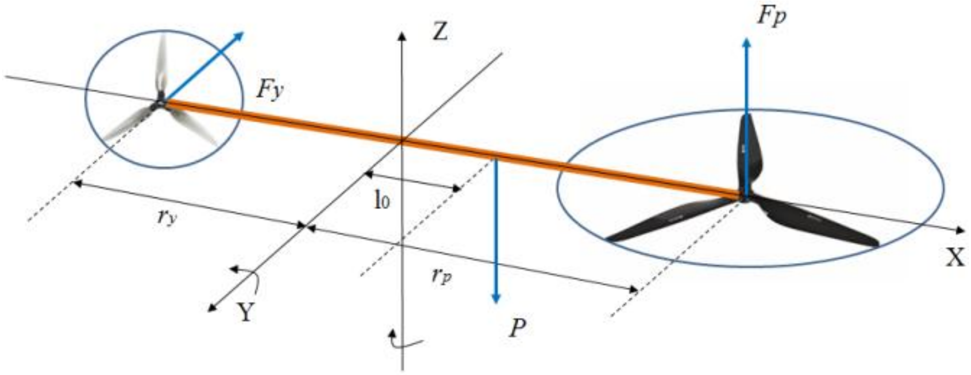

Fig. 1. Schematic of the TRMS

The TRMS is considered a rigid body, and the equations of motion are found by using the Euler-Lagrange equation. Based on studies [11], [13], [49]-[51], the following equations for the dynamics of the TRMS are show in (2):

$$\begin{cases} \ddot{\theta} = \dfrac{-mgl_0\cos(\theta) - B_p\dot{\theta} - mgl_0^2\dot{\alpha}^2\sin(\theta)\cos(\theta)}{J_{eq\_p} + ml_0^2} + \dfrac{K_{pp}V_p + K_{py}V_y}{J_{eq\_p} + ml_0^2} \\ \ddot{\alpha} = \dfrac{-B_y\dot{\alpha} + 2ml_0^2\dot{\alpha}\dot{\theta}\sin(\theta)\cos(\theta)}{J_{eq\_y} + ml_0^2\cos(\theta)^2} + \dfrac{K_{yp}V_p + K_{yy}V_y}{J_{eq\_y} + ml_0^2\cos(\theta)^2} \end{cases} \tag{2}$$

Where θ and α are the pitch and yaw angles, respectively; V_p and V_y are the voltages of the main and tail motors, respectively; B_p and B_y resistance coefficient of motion resistance acting on the pitch axis and yaw axis, respectively; K_{pp} and K_{yy} are the increase in torque from the main and tail motors; K_{py} is the increase in cross torque thrust acting on angle from the tail motor; K_{yp} is the increase in cross torque thrust impact on the deflection from the main motor. Note that the equivalent moment of inertia of the deflection will change as the pitch angle changes, but this value is assumed to be small and is not included.

Let $\mathbf{x_1} = [x_{11}, x_{12}]^T = [\theta, \alpha]^T$và $\mathbf{x_2} = [x_{21}, x_{22}]^T = [\dot{\theta}, \dot{\alpha}]^T$. According to the TRMS system, changing mass have little effect on the yaw angle. Therefore, the system can be converted to a general MIMO form with uncertain mass as follows [13]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = M + Nu + \Delta m\,[f(x_1, x_2) \quad 0]^T \end{cases} \tag{3}$$

Where $u = [V_p, V_y]^T$ represents the control input, Δm is the unknown component of the model parameters. Furthermore, N and M are determined from equation (3) as follows:

$$M = \begin{bmatrix} \dfrac{-mgl_0\cos(x_{11}) - B_px_{21} - mgl_0^2x_{22}^2\sin(x_{11})\cos(x_{11})}{Jeq_p + ml_0^2} \\ \dfrac{-B_yx_{22} + 2ml_0^2x_{22}x_{21}\sin(x_{11})\cos(x_{11})}{J_{eq_y} + ml_0^2\cos(x_{11})^2} \end{bmatrix}; \tag{4}$$

$$N=\begin{bmatrix}\dfrac{K_{pp}}{Jeq\_p+ml_0^2} & \dfrac{K_{py}}{Jeq\_p+ml_0^2}\\ \dfrac{K_{yp}}{J_{eq\_y}+ml_0^2\cos(x_{11})^2} & \dfrac{K_{yy}}{Jeq\_y+ml_0^2\cos(x_{11})^2}\end{bmatrix};$$

$$f(x)=-gl_0\cos(x_{11})(1+l_0x_{22}^2\sin(x_{11})).$$

2.2. Procedure for Synthesizing Sliding Mode Control Law based on Synergetic Control Theory

When it is not possible to immediately clearly specify a manifold with a sliding surface to synthesize a nonlinear SMC in synergetic control theory, a series or parallel set of invariant manifolds in the phase space of the system can be used based on developed scalar and vector synthesis [17], [19]-[30]. The sliding control synthesis method based on a sequential set of invariant manifolds was presented in the study [17], [19], [23]. Suppose the initial differential equation of the control object has the form:

$$\begin{cases}\dot{x}_i(t)=f_i(x_1,\dots,x_n)+a_{i+1}x_{i+1}, & i=\overline{1,n-1}\\ \dot{x}_n(t)=f_n(x_1,\dots,x_n)+u\end{cases} \tag{5}$$

Where $\mathbf{x}=[x_1, ...,x_n]^T$ is a vector of state variables, dim $\mathbf{x}$=n×1; u=u($\mathbf{x}$) is scalar control signals, $f_i(x_1, ...,x_n)$ $i=\overline{1,n-1}$, are continuously differentiable functions. In the first step of the synthesis, a manifold is considered.

$$\psi_1=\sum_{k=1}^{n-1}\beta_k|x_k|+|s_1|=0 \tag{6}$$

With $s_1=x_n-\varphi(x_1,...,x_n)=0$, where $\varphi(x_1,...,x_n)$ is an unknown continuous function at this stage, acting as an internal control for the decomposition system of the next stage:

$$\begin{cases}\dot{x}_i(t)=f_i(x_1,\dots,x_n)+a_{i+1}x_{i+1}, & i=\overline{1,n-2}\\ \dot{x}_{n-1}(t)=f_{n-1}(x_1,\dots,x_{n-1})+a_n\varphi_1(x_1,\dots,x_{n-1})\end{cases} \tag{7}$$

Based on the objective function equation of synergetic control theory and ADAR method.

$$T_1\dot{\psi}_1+\psi_1=0 \tag{8}$$

From the original equations of object (5), manifold (6) and (8) find the desired control law.

$$\begin{aligned}\varphi_1=&-\sum_{k=1}^{n-1}\beta_k(f_k+a_{k+1}x_{k+1})sign(x_k)sign(s_1)-\frac{1}{T_1}\psi_1 sign(s_1)\\ &-\sum_{i=1}^{n-1}\frac{\partial\varphi_1}{\partial x_i}(f_i+a_{i+1}x_{i+1})-f_n.\end{aligned} \tag{9}$$

This control signal transfers the system from an arbitrary initial state to a manifold $\psi_1=0$. Since the solution $\psi_1=0$ of equation (8) is asymptotically stable when $T_1>0$, this means that the system state definitely falls on the submanifold s_1 (on the sliding surface). Steady motion along $s_1=0$ can be organized using submanifolds $s_2,..., s_{n-1}$:

$$s_2=x_{n-1}-\varphi_2(x_1,\dots,x_{n-2})=0;\dots;s_{n-1}=x_2-\varphi_{n-1}(x_1)=0; \tag{10}$$

And synthesize the control laws $\varphi_2,..., \varphi_{n-1}$ based on the equations:

$$T_i\dot{s}_i + s_i = 0, \quad i = \overline{2, n-1};\ T_i > 0 \tag{11}$$

And the corresponding decomposition system has the form (7). For MIMO systems, the first step is to determine the sliding manifold for each channel and then solve the system of equations to find the control signal for each channel. With this control law, the system is guaranteed to be internally stable from the channel and the next step uses the submanifolds s_2, .., s_{n-1} as above with n being the degrees of freedom in each channel.

2.3. Synthesis of Sliding Mode Control for a TRMS based on Synergetic Control Theory

The purpose of the TRMS control problem is to ensure that the pitch and yaw channels move in the desired trajectory $\mathbf{x}_{sp}$ by changing the voltage supplied to the two motors to create torque acting on the two channels.

From the perspective of the synergetic control theory, this means that it is necessary to synthesize the control signal u(x_1, x_2, x_3, x_4) (SiSMC) - a function that depends on the phase coordinates. The control signal will move the TRMS position from the initial position following a given signal or stabilize at the desired position when there is disturbance to ensure the required quality of the system.

From the control problem, based on the synergetic control theory for technical systems, the first technological invariant corresponding to the control objective is presented:

$$x_1 = x_{sp} = [\theta_{sp} \quad \alpha_{sp}]^T \tag{12}$$

In the first step, based on the mathematical model, when the control vector $\boldsymbol{u}$ changes, it will affect the dynamics of the pitch and yaw channels. The first manifold chosen is of the form:

$$\psi_1 = [|s_{11}| \quad |s_{21}|]^T = [0 \quad 0]^T \tag{13}$$

Where $s_{11}=x_{21}-\varphi_{11}(x_{11},\theta_{sp})$, $s_{21}=x_{22}-\varphi_{21}(x_{12},\alpha_{sp})$, the function $\varphi_{11}(x_{11},\theta_{sp})$, $\varphi_{21}(x_{12},\alpha_{sp})$ determine the desired characteristics of the change. Changing the angular velocities of the pitch and yaw channels at the intersection with the invariant manifold $\boldsymbol{\psi}_1$. The functions $\varphi_{11}(x_1,\theta_{sp})$, $\varphi_{21}(x_{12},\alpha_{sp})$ are determined in the process of synthesizing the control law, proceeding from the invariant condition (12). To ensure the manifold (13) is globally stable, according to the analytical design method of synthetic regulators (ADAR) [17]-[22], the macro variable $\boldsymbol{\psi}_1$ must satisfy the solution of the basic functional equation:

$$T_1\dot{\psi}_1 + \psi_1 = 0 \tag{14}$$

Where $T_1 = \begin{bmatrix} T_{11} & 0 \\ 0 & T_{22} \end{bmatrix}$ is a positive definite diagonal matrix that ensures the conditions for asymptotic stability of the system's channels. Substituting equation (13) into (14), the control vector $\boldsymbol{u}$ is found as follows.

Where $S = \begin{bmatrix} sgn(s_{11}) & 0 \\ 0 & sgn(s_{21}) \end{bmatrix}$. When the system enters the manifold, the system's representation point touches the intersection of the manifold $s_{11}=0$ and $s_{21}=0$, and then the system will undergo dynamic decomposition of the subsystems in (4). After that, the dynamics of the subsystems according to the channels are described by the equations on the Pitch channel and on the Yaw channel (16).

Where the functions $\varphi_{11}(x_{11}, \theta_{sp})$ and $\varphi_{21}(x_{12}, \alpha_{sp})$ in the decomposition system (16) are considered internal control signals. In the 2nd step of the synthesis, to find the control law and to determine the functions $\varphi_{11}(x_{11}, \theta_{sp})$ and $\varphi_2(x_{12}, \alpha_{sp})$, an additional invariant manifold is introduced, which will ensure the stability of the closed-loop system and the fulfillment of technological invariance (12). Dynamic system (16) ensures stability and satisfy technological invariance will choose the internal control signal $\varphi_{11}(x_{11})=-k_{11}(x_{11}-\theta_{sp})$ and $\varphi_{21}(x_{12})=-k_{21}(x_{12}-\alpha_{sp})$. So the equations of the decomposition system have the following form:

$$\boldsymbol{u}=\boldsymbol{N}^{-1}\left(\boldsymbol{-M}-\boldsymbol{T}_1^{-1}\boldsymbol{S}^{-1}\boldsymbol{\psi}_1+\begin{bmatrix}\dfrac{\partial\varphi_{11}}{\partial x_{11}}x_{21}+\dfrac{\partial\varphi_{11}}{\partial\theta_{sp}}\dot{\theta}_{sp}\\ \dfrac{\partial\varphi_{21}}{\partial x_{12}}x_{22}+\dfrac{\partial\varphi_{j,1}}{\partial\alpha_{sp}}\dot{\alpha}_{sp}\end{bmatrix}\right) \tag{15}$$

$$\dot{x}_1=[\varphi_{11}(x_{11},\theta_{sp})\quad \varphi_{21}(x_{12},\alpha_{sp})]^T \tag{16}$$

$$\dot{x}_1=\left[-k_{11}\left(x_{11}-\theta_{sp}\right)\quad -k_{21}\left(x_{12}-\alpha_{sp}\right)\right]^T \tag{17}$$

For the equations (17) to be asymptotically stable at $x_{11}=\theta_{sp}$ and $x_{12}=\alpha_{sp}$, the condition is that k_{11} and k_{21} are positive constants. From the equation (15) and the manifolds $\varphi_{11}(x_{11}, \theta_{sp})$ and $\varphi_{21}(x_{12}, \alpha_{sp})$ selected above, the control vector **u** for TRMS is found:

$$u=N^{-1}\left(-M-T_1^{-1}S^{-1}\psi_1+\begin{bmatrix}-k_{11}x_{21}+k_{11}\dot{\theta}_{sp}\\ -k_{21}x_{22}+k_{21}\dot{\alpha}_{sp}\end{bmatrix}\right) \tag{18}$$

2.4. Synthesis of Adaptive Sliding Mode Control based on Synergetic Control Theory

During operation the mass of TRMS may not be known accurately or may change. Control based on conventional sliding control and sliding mode control based on synergetic control theory do not guarantee control qualities. A common approach to robust control is to design the control law for control systems affected by bounded disturbances. But in this section, the authors design an adaptive controller combined with estimated values of uncertainty in the control law. According to the ADAR method from equation (13), when the system has an uncertain component, the proposed control law (ASiSMC) is as follows:

$$u_{adap.}=N^{-1}\left(-M-\begin{bmatrix}\Delta\widehat{m}f(x)\\ 0\end{bmatrix}-T_1^{-1}S^{-1}\psi_1+\begin{bmatrix}-k_{11}x_{21}+k_{11}\dot{\theta}_{sp}\\ -k_{21}x_{22}+k_{21}\dot{\alpha}_{sp}\end{bmatrix}\right) \tag{19}$$

Where $\Delta\widehat{m}$ is the estimated value of Δm. Consider the Lyapunov function:

$$V=0.5[s_{11}\quad s_{21}][s_{11}\quad s_{21}]^T \tag{20}$$

Differentiating the function (20) with the control law (19) affecting the system (3):

$$\dot{V}=[s_{11}\quad s_{21}]\left(\Delta\widetilde{m}\begin{bmatrix}f(x)\\ 0\end{bmatrix}-T_1^{-1}S^{-1}\psi_1\right)=-\psi_1^T T_1^{-1}\psi_1+[s_{11}\quad s_{21}]\Delta\widetilde{m}\begin{bmatrix}f(x)\\ 0\end{bmatrix} \tag{21}$$

Where $\Delta\widetilde{m}=\Delta m-\Delta\widehat{m}$ is the error between the uncertainty mass and the estimated uncertain mass. To design an adaptation control law, choose a Lyapunov function of the following form:

$$V_{ad}=V+\frac{1}{2}\gamma\Delta\widetilde{m}^2 \tag{22}$$

Where γ is a positive constant. Taking the time derivative of the Lyapunov function and assuming the values of the uncertainty masses are constant leads to. The adaptive control law is built based on expression (23). With this proposed adaptive control law, the time derivative of the Lyapunov function (22).

$$\dot{V}_{ad} = -\psi_1^T T_1^{-1}\psi_1 + [s_{11} \quad s_{21}]\Delta\tilde{m}\begin{bmatrix} f(x) \\ 0 \end{bmatrix} + \gamma\Delta\tilde{m}\Delta\dot{\hat{m}} \tag{23}$$

$$\Delta\dot{\hat{m}} = -\gamma^{-1}gl_0\cos(x_{11})(1 + l_0x_{22}^2\sin(x_{11}))s_{11}; \tag{24}$$

$$\dot{V}_{ad} = -\psi_1^T T_1^{-1}\psi_1 \le 0 \tag{25}$$

According to Barbalet's theorem, $\boldsymbol{\psi}_1 \to 0$ when $t\to\infty$, combined with the ADAR method from the system of equations (13) leads to an asymptotically stable system.

2.5. Synthesis of Sliding Mode Control for a TRMS

The sliding mode control law is synthesized based on research [8]. From the equations (3), it is necessary to determine the control signal to achieve the desired outputs (12). The outputs of the system:

$$e_1 = [x_{11} - \theta_{sp} \quad x_{12} - \alpha_{sp}]^T \tag{26}$$

Select the sliding surface of the controller as follows:

$$s = \dot{e}_1 + \lambda e_1 \tag{27}$$

Where $\lambda = \begin{bmatrix} \lambda_{11} & 0 \\ 0 & \lambda_{22} \end{bmatrix}$ is a positive definite diagonal matrix that ensures the condition for asymptotic stability of the sliding surface. Applying sliding mode control theory:

$$\dot{s} = Ksign(s) \Rightarrow (\dot{x}_2 - \ddot{x}_{sp}) + \lambda(\dot{x}_1 - \dot{x}_{sp}) = Ksgn(s) \tag{28}$$

Where $K = \begin{bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{bmatrix}$ with K_{11}, K_{22} are positive constants. Then, the sliding mode control law (SMC) is determined as follows:

$$u = N^{-1}\left(-M - \lambda(\dot{x}_1 - \dot{x}_{sp}) + \ddot{x}_{sp} - Ksat(s)\right) \tag{29}$$

In formula (29), the saturation function $sat(\boldsymbol{s})$ is chosen instead of the $sign(\boldsymbol{s})$ function to reduce the chattering phenomenon that can damage the actuator.

3. Results and Discussion

The simulation results were performed to demonstrate the potential of the proposed control law for TRMS. The parameters of the model in this study are as follows: J_{eq_p} =0.0215 (kg·m^2); Jeq_y= 0.0237(kg·m^2), M=2.0 (kg), B_p=0.0071(N/V); B_y=0.0220 (N/V), l_0=0.002 (m), K_{pp}=0.022 (N·m/V); K_{py}=0.0221(N·m/V), K_{yp}=-0.0227(N·m/V), K_{yy}=0.0022 (N·m/V), g=9.8(m/s2). The proposed control law parameters are selected as follows: T_{11}=0.1, T_{22}=0.25, k_{11}=50, k_{21}=50; adaptive law parameters $\gamma_{1,1}$=10, $\gamma_{2,2}$=20 and sliding controller parameters λ_{11}=10, λ_{22}=4, K_{11}=20, K_{22}=20. The simulations of the three controllers are conducted with 2 scenarios: the scenario 1 uses the desired signal $\mathbf{x}_{sp}$ as a step function with different values in a period of time 10(s); the scenario 2 is when the initial state of the origin system will track the orbit with the desired tracking signal of the form $\mathbf{x}_{sp}(t)=[1.4\sin(t);\ 1.4\cos(t)]^T$. In both of the above scenarios, the object model has a changing mass at each point in time, as shown in Fig. 2 and Fig. 4.

In the scenario 1, the responses of angle of pitch and yaw are shown in Fig. 2 and Fig. 3 with the control laws proposed above. A quality control process with several indicators is shown in Table 1. We can see that, in the period 0-5 (s) when the mass of the system is the same as the initial mass, all three controllers give good results, although ASiSMC has a higher overshoot (28%) but still ensures the system operates stably and the static error reaches 0 in less than 1 (s). With TRMS, this overshoot does not exceed technical conditions. During the period of 5-10 (s), 15-20 (s) when the mass changes greatly, the SiSMC law and SMC law (0.78 rad.) both have quite large static errors and cannot eliminate that error. Therefore, the position of TRMS cannot be stabilized in the desired position. For the ASiSMC rule, the static error is 0, and the response time and overshoot are very small. This proves the effectiveness of the adaptive law (24) when the mass changes. During the period of 10-15(s) when the set value changes from 0.4 (rad.) to - 0.3 (rad.) and the mass decreases by 14 (kg), it shows that the SiSMC and SMC laws still have small static errors. The ASiSMC control law has overshoot (29%), but the static error is 0 and the settling time is as small as in the first stage. For the Yaw angular channel, its response is shown in Fig. 3. The results show that the Yaw angle response for all three proposed laws is equivalent, with no difference in the setup time. During the transition period, although there is a difference, it is not significant. This shows that, in the design process of the three control laws, the cross-session component between the two channels is considered.

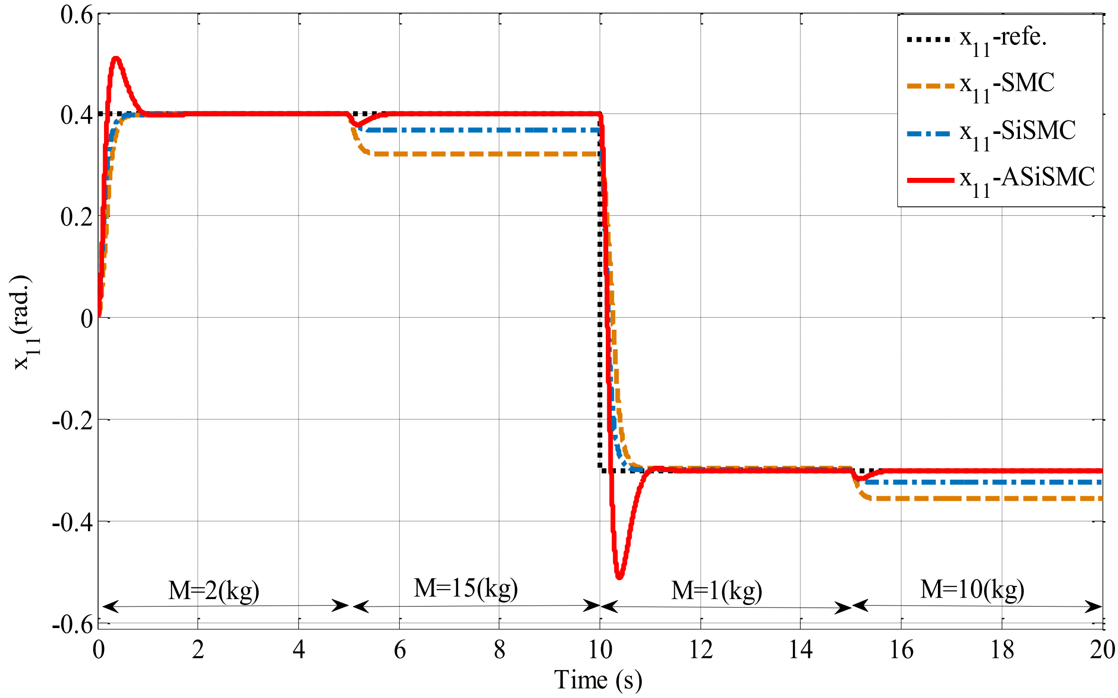

Fig. 2. Pitch angle response of twin rotor MIMO system with scenario 1

In the scenario 2, the pitch and yaw response of the TRMS with the three control laws given above are shown in Fig. 4, Fig. 5 and Fig. 6. From graphs 4 and 5 we can see that with a set signal in the form of harmonic oscillation, the quality of the 3 control laws all give small tracking errors, but at the beginning the SiSMC and ASiSMC control laws have errors larger than SMC. Because in the two control laws (18) (19) has an initial value of 0 while the control law SMC (29) has an initial value other than 0.

SMC gives small tracking error in the range [-0.005; 0.005] (rad.) when the system mass is close to the initial mass (0-5 (s), 10-15 (s)), but the error is large [-0.08; 0.08] (rad.) when there is a change in mass (5-10 (s), 15-20 (s)). SiSMC gives errors in the range [-0.045; 0.045] (rad.), while ASiSMC is in the range [-0.0215; 0.0215] (rad.) when the mass parameter changes. SiSMC gives errors in the range [-0.045; 0.045] (rad.), while ASiSMC is in the range [-0.0215; 0.0215] (rad.) when the mass parameter changes. The Yaw angle channel in Fig. 6, it shows that only in the early stages, the ASiSMC and SiSMC laws respond faster than SMC, while during the control process,

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the system quality is not significantly different. This means that the cross-link compensation component in the three control laws works effectively in both scenarios.

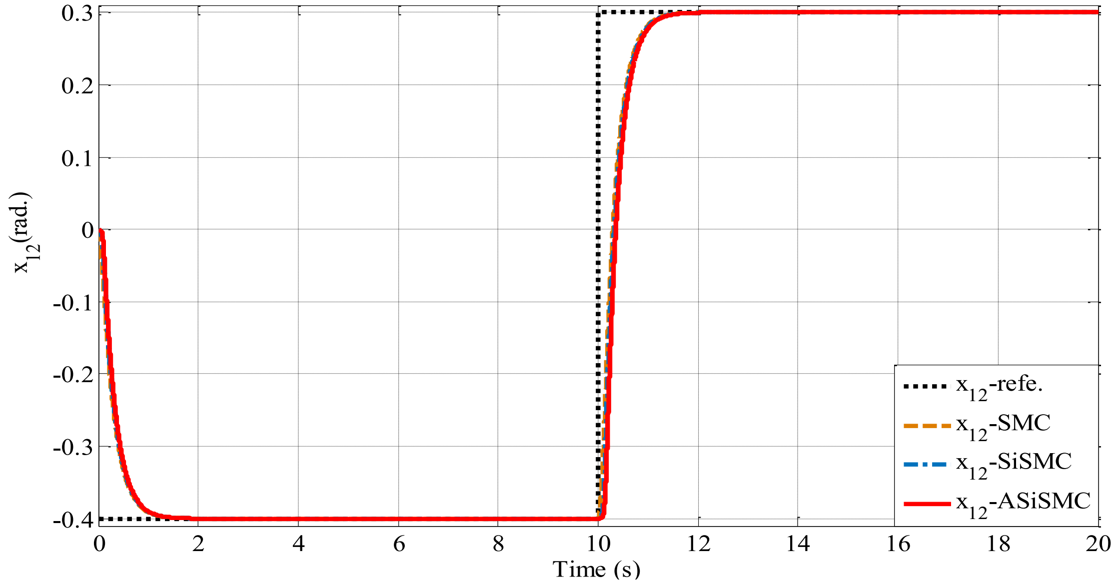

Fig. 3. Yaw angle response of twin rotor MIMO system with scenario 1

Table 1. Comparison of the quality of the three controllers on the pitch angle channel

Variable	Time (0-5s) (10-15s)			Time (5-10s) (15-20s)		
	SMC	SiSMC	ASiSMC	SMC	SiSMC	ASiSMC
Static Error (rad.)	0- 0.078	0 -0.031	0 0	0.004 -0.056	0.002 -0.023	0 0
Settling Time (s)	0.43 ∞	0.35 ∞	0.75 0	0.52 ∞	0.37 ∞	0.745 0
Overshoot (%)	0 19.5	0 7.8	28 0	0 18.7	0 7.7	29 0

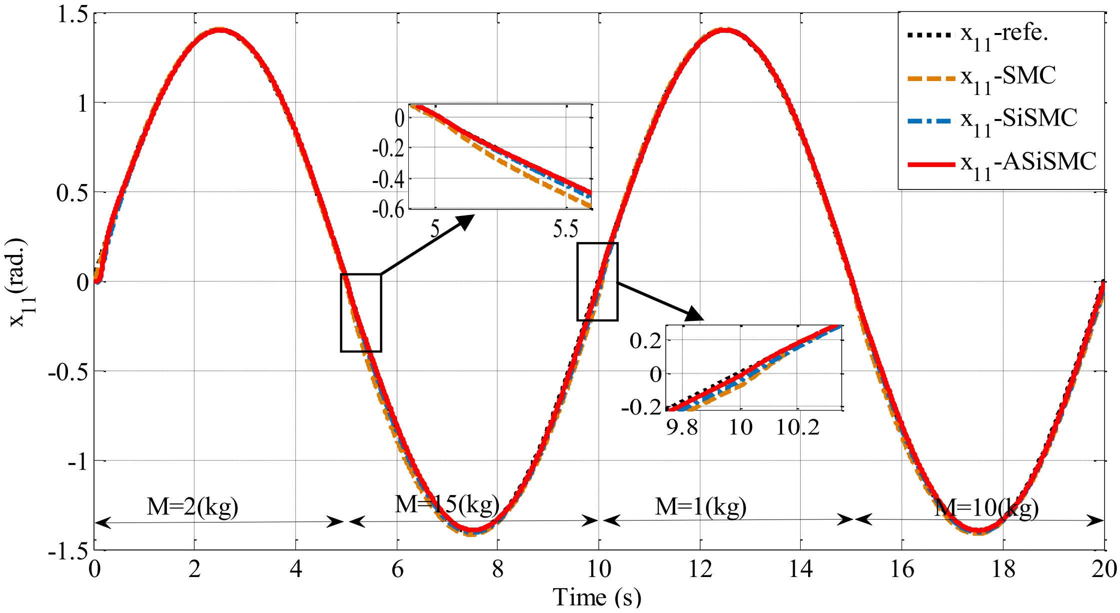

Fig. 4. Pitch angle response of twin rotor MIMO system with scenario 2

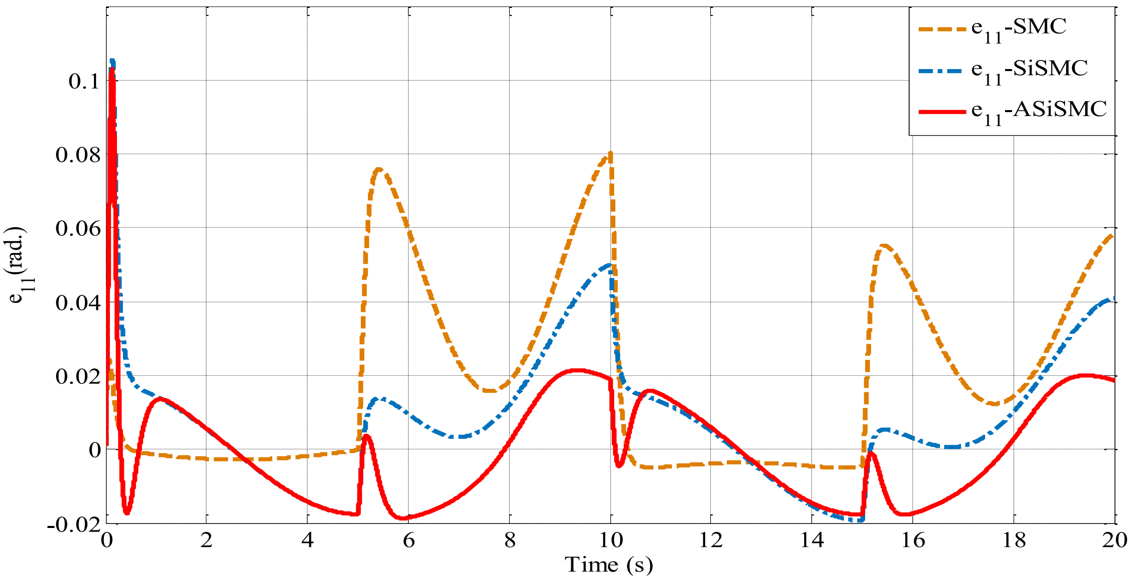

Fig. 5. Pitch channel tracking error $e_{11}= \theta_{sp}-x_{11}$ with scenario 2

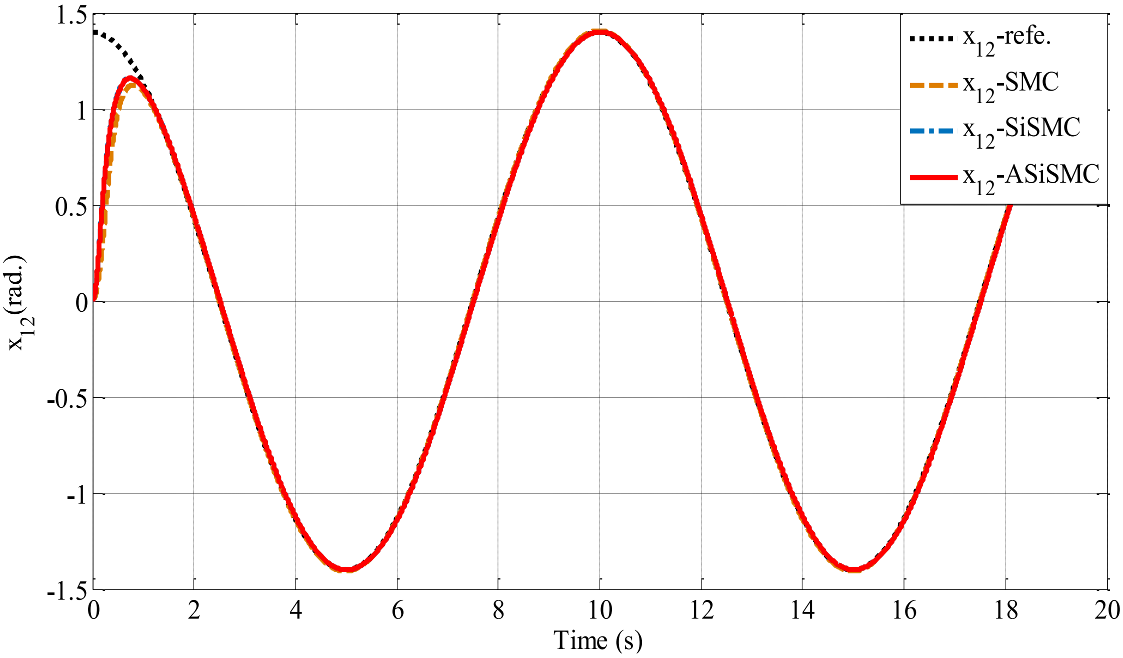

Fig. 6. Yaw angle response of Twin Rotor MIMO System with scenario 2

4. Conclusion

This paper presents a method to build an adaptive sliding mode control based on the synergetic control theory of the mass parameters for Twin Rotor MIMO System. According to the simulation results, the proposed controller shows a good response to different types of desired signals. The SMC gives the poorest response quality. The sliding mode control based on the synergetic control theory gives a highly stable response to uncertain parameters but does not eliminate static errors. The adaptive sliding mode control based on the synergetic control theory ensures system stability when there are changing masses. Furthermore, the study also proves the stability of the control law through the Lyapunov function. Finally, the simulation results have verified the superiority and effectiveness of the SiSMC and ASiSMC proposed in this study. In the future, we will focus on control research with manifolds that consider the cross-linking between the 2 channels, which means considering the physical properties of the manifold. In addition, neural networks will be used while designing an adaptive controller based on Synergetic Control Theory when there are external disturbances and many uncertain parameters.

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