Control Systems and Optimization Letters, Vol. 3, No 1, 2025
ISSN: 2985-6116, DOI: 10.59247/csol.v3i1.169

Cascade Control for Trajectory-Tracking Mobile Robots Based on Synergetic Control Theory and Lyapunov Functions

Nguyen Xuan Chiem
Department of Automation and Computing Techniques, Le Quy Don Technical University, Vietnam
Email: chiemnx@mta.edu.vn
*Corresponding Author

***Abstract*—This paper aims to synthesize a synergetic control law using a cascade approach for trajectory-tracking robots. A nonlinear model was established for a differential two-wheeled mobile robot. The robot's operation can either stabilize along a desired trajectory or deviate due to model uncertainty and external disturbances. The cascade approach is utilized to reduce system complexity while maintaining the robustness of the control law. The kinematic control law in the outer loop is designed using Lyapunov functions, while the dynamic control law is derived using synergetic control theory. This law ensures system control quality under model uncertainties and external disturbances. Finally, simulation results demonstrate that the proposed controller provides robust stability for the mobile robot, along with excellent disturbance rejection and robustness against model uncertainties.**

***Keywords*—Synergetic Control Law, Lyapunov Functions, Mobile Robot, Trajectory-Tracking Robot, Cascade Approach**

I. INTRODUCTION

Wheel-based mobile robots have been extensively studied and applied worldwide in recent years. This field attracts significant attention from researchers due to the low energy consumption, low mechanical complexity, fast mobility, and intelligent motion capabilities independent of human intervention. Such robots are particularly beneficial in replacing humans in dangerous tasks, such as explosive material detection, transporting goods in hazardous environments, security monitoring, and various other fields. Given the increasing applications of mobile robots in hazardous or unpredictable environments, such as search-and-rescue missions or autonomous vehicles, ensuring reliable trajectory tracking under real-world conditions is crucial. The proposed synergetic control framework promises a robust solution to meet these challenges.

The navigation and control of wheel-based mobile robots have been studied more intensively in recent years due to their applications in challenging situations with evolving control techniques. Differential-drive mobile robots are commonly used in robotic research. Although the mathematical model of such robots appears simple, the existence of nonlinear factors and constraints makes controlling these systems challenging. Many studies on stabilization and trajectory tracking have been published [1]-[13]. Studies [1]-[3] employed the Lyapunov method to design control laws for two-wheeled and four-wheeled mobile robots. The control laws are chosen to ensure that the Lyapunov function remains positive and its derivative is always negative. For wheel-based robots, the control variables are typically the robot's linear and angular velocities or the individual wheel velocities. Backstepping control methods are presented in [4], [5]. These results demonstrate the effectiveness of proposed algorithms in addressing nonholonomic constraints and achieving satisfactory performance. Model-based predictive control (MPC) for mobile robots is discussed in [6], [7]. Trajectory tracking for autonomous vehicles is typically solved by designing control laws that enable the vehicle to follow predefined feasible trajectories based on trajectory errors. In [8], sliding mode controllers were designed to account for disturbances. Results show high performance in trajectory tracking even with external disturbances and parameter uncertainties. For mobile robots with uncertain model parameters, adaptive control laws were proposed in [9], [10]. The dynamic parameters of the robot are updated online, reducing errors and enhancing performance in applications where parameters vary, such as transporting loads. The stability of the entire system is analyzed using Lyapunov theory, and the control errors are proven to be ultimately bounded. Controllers based on fuzzy logic and neural networks are discussed in [11]-[14]. Fuzzy controllers are widely used in mobile robot applications involving obstacle avoidance, path planning, and autonomous navigation. Establishing relationships between control law parameters and system state variables without requiring specific models has made proposed motion control systems simple, easy to implement, and effective compared to traditional control systems.

The studies reviewed above employed intelligent control schemes based on fuzzy logic, neural networks, nonlinear control such as sliding mode control, predictive control, backstepping control, or adaptive control. Furthermore, it should be noted that the structure of the controller varies across studies. Unlike previous studies, this paper develops a highly robust controller for trajectory tracking of mobile robots based on synergetic control theory and a cascade approach. Synergetic control (SC), grounded in state-space theory, is used to design and control highly complex nonlinear systems. This control strategy enables system state variables to evolve on designer-specified invariant manifolds, achieving desired performance despite the presence of uncertainties and disturbances [15]-[19]. Although several control methods, including Lyapunov-based control, sliding mode, and model predictive control, have been explored, they often face challenges when dealing with model uncertainties and external disturbances. To address these limitations, this paper proposes the use of synergetic control theory, an advanced approach that ensures robustness and precision in trajectory tracking under complex conditions.

This paper investigates the design of synergetic control laws for trajectory-tracking mobile robots based on kinematic and dynamic models. A suitable Lyapunov function is chosen for designing the control law for the kinematic layer. The dynamic control law is derived based on synergetic control theory, ensuring the robot tracks the desired trajectory with permissible small errors. Additionally, the cascade control structure ensures that the system tracks the desired trajectory under parameter uncertainties and external disturbances. Unlike traditional control methods, which may struggle with model uncertainties or disturbances, this paper introduces a robust and effective solution by combining synergetic control theory with a cascade approach. This integration enables superior trajectory tracking performance even in the presence of external disturbances and model variations.

The paper is organized as follows. Section 2 describes the mathematical model of the mobile robot. Section 3 provides an overview of synergetic control and the synthesis of control laws based on the cascade control architecture. Section 4 presents simulation results and discussions with the proposed control law. Finally, conclusions and future research directions are provided in Section 5.

II. MATHEMATICAL MODEL OF LINE-TRACKING ROBOT

To describe the state of a mobile robot, two reference frames need to be defined: the global reference frame $Ox_Iy_Iz_I$ and the robot's local reference frame $Ax_ry_rz_r$.

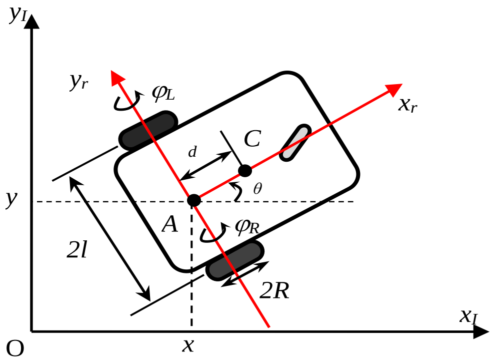

Fig. 1. Wheeled mobile robot

Fig. 1 illustrates the model of a wheeled mobile robot. The configuration includes a robot chassis with two fixed wheels on a shaft driven by identical DC motors and a front caster wheel. R is the radius of the two wheels, C is the robot's center of mass, A is the midpoint of the axle connecting the centers of the two wheels, $2l$ is the distance between the two wheels, φ_R and φ_L the speeds of the right and left wheels, d is the distance between A and C. The angle between the axes Ax_r and Ox_I is denoted as θ.

The state of the robot in the reference frame is represented by the position of point A, which is located at the midpoint of the wheel axis, and the orientation angle of the robot θ. Two different coordinate systems were designated as the Inertial Coordinate System $\boldsymbol{q^I} = [x\ y\ \theta]^T$ and Robot Coordinate System $\boldsymbol{q^r} = [x_r\ y_r\ \theta_r]^T$, respectively. Robot position matrices in the robot coordinate system and inertial coordinate system respectively and orthogonal rotation matrix were defined as below $\boldsymbol{q^I} = \boldsymbol{R}(\theta)\boldsymbol{q^r}$, where $\boldsymbol{R}(\theta)$ rotates the robot's frame.

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

A. Nonholonomic Constraints

Nonholonomic constraints mean that the robot's movement is restricted in certain directions due to the mechanics of its wheels. Two primary constraint equations:

In Equation (2), clarify that it describes how the robot is constrained to move along a flat surface, implying no vertical movement or energy change.

$$-\dot{x}\sin\theta + \dot{y}\cos\theta = 0 \tag{2}$$

In Equation (3), explain the rolling without slipping condition in simpler terms. This ensures the robot can only move forward/backward without lateral sliding.

$$\begin{aligned} \dot{x}\cos\theta + \dot{y}\sin\theta + b\dot{\theta} - R\dot{\varphi}_R &= 0 \\ \dot{x}\cos\theta + \dot{y}\sin\theta - b\dot{\theta} - R\dot{\varphi}_L &= 0 \end{aligned} \tag{3}$$

B. Kinematic Model

Let v and ω represent the linear velocity and angular velocity of the robot in the local reference frame. The relationship between the robot's workspace velocities and joint space velocities can be expressed in matrix form:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = R(\theta)\begin{bmatrix} v \\ 0 \\ \omega \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} v \\ 0 \\ \omega \end{bmatrix} \tag{4}$$

The linear velocity and angular velocity of the mobile robot in the robot reference frame are the average values of the linear velocities of the two wheels in matrix form as follows:

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = A_k \begin{bmatrix} \dot{\varphi}_R \\ \dot{\varphi}_L \end{bmatrix} \tag{5}$$

With

$$A_k = \begin{bmatrix} \frac{R}{2} & \frac{R}{2} \\ \frac{R}{2l} & -\frac{R}{2l} \end{bmatrix} \tag{6}$$

Combining (8) and (9), the velocity of DDWMR in the reference frame is as follows:

$$\dot{q}^I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{R}{2}\cos(\theta) & \frac{R}{2}\cos(\theta) \\ \frac{R}{2}\sin(\theta) & \frac{R}{2}\sin(\theta) \\ \frac{R}{2l} & -\frac{R}{2l} \end{bmatrix}\begin{bmatrix} \dot{\varphi}_R \\ \dot{\varphi}_L \end{bmatrix} \tag{7}$$

Equation (7) represents the forward kinematic model of the mobile robot. It shows the relationship between the velocity components of the robot and the speed of the wheels.

C. Dynamic Model

The dynamic model of a mobile robot, based on Lagrange mechanics, is expressed as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = F - A^T(q)\lambda \tag{8}$$

Where $L = T - V$ is the Lagrangian function, T is the kinetic energy of the system, V is the potential energy of the system, $\boldsymbol{q} = \left[x\ y\ \theta\ \varphi_R\ \varphi_L\right]^T$ are the general coordinates, $\boldsymbol{F} = [0\ 0\ 0\ \tau_R\ \tau_L]^T$ is the global force vector, τ_R, τ_L are the moments applied to the wheels, $\boldsymbol{A^T(q)}$ is the constraint matrix and $\boldsymbol{\lambda}$ is the factor vector Lagrangian association with nonholonomic constraints. Under the assumption that the robot's operation satisfies the constraints in section II.A. For the convenience of control and simulation purposes, the general equation of motion (8) is converted into an alternative form, the main purpose of which is to eliminate the term representing the constraint forces $\boldsymbol{A^T(q)}$. From this, we obtain the mobile robot dynamics model as follows [12]-[15]:

$$\begin{cases} \left(m + \dfrac{2I_w}{R^2}\right)\dot{v} - m_c d\omega^2 + d_1 = \dfrac{1}{R}\tau_v \\ \left(I + \dfrac{2l^2 I_w}{R^2}\right)\dot{\omega} + m_c d v\omega + d_2 = \dfrac{l}{R}\tau_\omega \end{cases} \tag{9}$$

Where: $m = m_c + m_\omega$ is the total mass of the mobile robot, m_c is the mass of the robot excluding the wheels and motors, m_ω is the mass of each driving wheel together with the motor, $I = I_c + m_c d^2 + 2m_\omega L^2 + 2I_m$ is the total equivalent inertia, I_c is the moment of inertia of the robot about the vertical axis passing through the center of mass C, I_ω is the moment of inertia of each driving wheel and motor with the wheel axis, I_m is the moment of inertia of each wheel and motor with the vertical axis passing through C, d_1 and d_2 are considered as the total disturbances including internal uncertainties and external disturbances affecting the linear velocity v and the angular velocity ω. $\tau_v = \tau_R + \tau_L$, $\tau_\omega = \tau_R - \tau_L$.

The system of equations (9) represents the complete dynamic model for the mobile robot, which is used to analyze and simulate the trajectory tracking motion control algorithm in the next section.

III. SYNTHESIS OF AN SYNERGETIC TRAJECTORY TRACKING CONTROLLER FOR A MOBILE ROBOT

A. Theoretical Foundation of Synergetic Control

The synthesis of an synergetic controller begins with the definition of a macroscopic variable, which is a function of the system's state variables:

$$\psi(t) = \psi\,(x, t\,) \tag{10}$$

The control objective is to constrain the system to operate on the manifold $\psi = 0$. The designer can choose the characteristics of this macroscopic variable based on control specifications (e.g., limits on control outputs, etc.). In the trivial case, ψ is a simple linear combination of state variables. This process is then repeated, defining as many macroscopic variables as there are control channels. Next, the dynamic evolution of these macroscopic variables is defined according to the equation:

$$T\dot{\psi}(t) + \psi(t) = 0; \quad T > 0 \tag{11}$$

Where T is a design parameter describing the rate of convergence to the manifold specified by the macroscopic variable. Finally, the control law (the time evolution of the control output) is synthesized using Equation (11) and the system's dynamic model.

In summary, any manifold introduces a new constraint on the domain of the state-space, thereby reducing the order of the system and guiding it towards global stability. The procedure outlined above can be easily implemented as a computer program for automatic control law synthesis or can be performed manually for systems such as mobile robot.

These results are obtained while working with the complete nonlinear system, and the designer does not need to make simplifications during the modeling process to derive a linear description, as is required in classical control theory.

B. Design of Trajectory Tracking Control Law for Mobile Robot

In this paper, the cascade control method is applied to divide the robot model into two layers: the outer layer is the robot's kinematic control loop based on the Lyapunov function, and the inner layer is the robot's dynamic control loop based on the synergetic control theory. The block diagram of the control system is shown in Fig. 2.

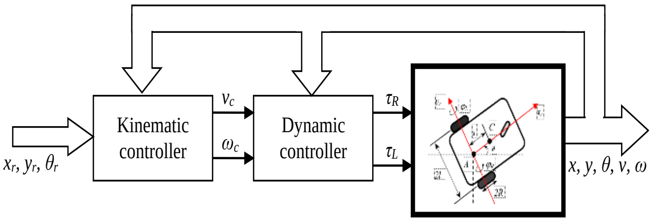

Fig. 2. Control structure diagram for mobile robot tracking

a) Kinematic Tracking Control Law

The input of the system is a vector consisting of the linear velocity and angular velocity of the reference robot. The output of the system is the current position of the robot. During the control process, there is always a position error between the desired position and the current position of the robot due to unknown disturbances and measurement errors. The kinematic control law computes the desired linear velocity and angular velocity of the robot. The kinematic controller plays an important role in driving these errors to 0.

In the proposed control system, two positions will be used: the current position $\boldsymbol{q} = [x\ y\ \theta]^T$ and the reference position $\boldsymbol{q_r} = [x_r\ y_r\ \theta_r]^T$. The pose error will be defined as follows:

$$e_p = \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{12}$$

The derivative of the error e_p is given by:

$$\begin{aligned}\dot{e}_x &= (\dot{x}_r - \dot{x})\cos\theta + (\dot{y}_r - \dot{y})\sin\theta - \\ &\quad (x_r - x)\dot{\theta}\sin\theta + (y_r - y)\dot{\theta}\sin\theta \\ &= \dot{x}_r\cos(\theta_r - \theta) + \dot{y}_r\sin(\theta_r - \theta) - v + e_y\omega \\ &= -v + e_y\omega + v_r\cos e_\theta\end{aligned} \tag{13}$$

$$\begin{aligned}\dot{e}_y &= -(\dot{x}_r - \dot{x})\sin\theta + (\dot{y}_r - \dot{y})\cos\theta \\ &\quad -(x_r - x)\dot{\theta}\cos\theta - (y_r - y)\dot{\theta}\sin\theta \\ &= -\dot{x}_r\sin(\theta_r - \theta) + \dot{y}_r\cos(\theta_r - \theta) - e_x\omega \\ &= -e_x\omega + v_r\sin e_\theta\end{aligned} \tag{14}$$

$$\dot{e}_\theta = \dot{\theta}_r - \theta = \omega_r - \omega \tag{15}$$

From equations (13), (14), and (15), the robot's kinematic system of equations is given by:

$$\begin{bmatrix}\dot{e}_x \\ \dot{e}_y \\ \dot{e}_\theta\end{bmatrix} = \begin{bmatrix}-v + e_y\omega + v_r\cos e_\theta \\ -e_x\omega + v_r\sin e_\theta \\ \omega_r - \omega\end{bmatrix} \tag{16}$$

From the system of equations (16), in order to drive the state errors to zero, we need to find the control laws v and ω such that the system of equations (16) is asymptotically stable.

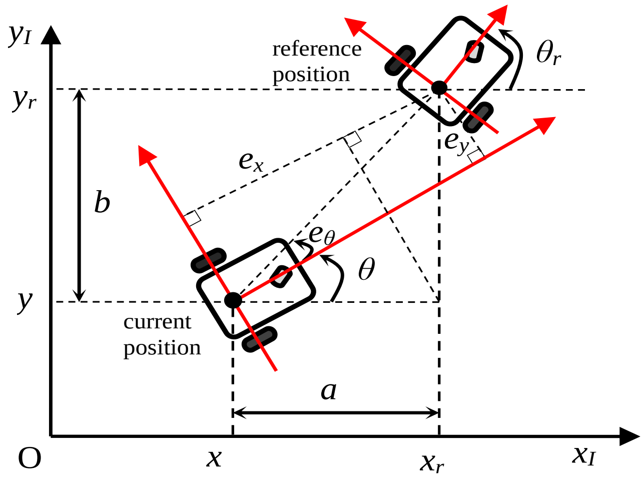

Fig. 3. Position tracking error of mobile robot

Consider the positive definite Lyapunov function V with $k_\theta>0$ and its derivative as follows:

$$V = \frac{1}{2}\left(e_x^2 + e_y^2\right) + k_\theta(1 - \cos e_\theta) \tag{17}$$

The derivative of the Lyapunov function is given by:

$$\begin{aligned}\dot{V} &= (-v + v_r\cos e_\theta)e_x \\ &+\left(e_y v_r + k_\theta\omega_r - k_\theta\omega\right)\sin e_\theta\end{aligned} \tag{18}$$

To make the system (16) asymptotically stable around the stable equilibrium point $\boldsymbol{e}_p = 0$, we choose the kinematic control law as follows:

$$\begin{cases} v_c = v_r\cos e_\theta + k_x e_x \\ \omega_c = \dfrac{1}{k_\theta}e_y v_r + \omega_r + \dfrac{k_y}{k_\theta}\sin e_\theta\end{cases} \tag{19}$$

With the kinematic control law (19), we have:

$$\dot{V} = -k_x e_x^2 - k_y(\sin e_\theta)^2 \le 0 \tag{20}$$

Thus, the derivative of V is a negative definite function. This means that by using the control law (19), the system will be asymptotically stable around the equilibrium point, with the conditions v_r, ω_r, being bounded and continuous, and k_x k_y k_θ being bounded and positive constants. The values v_c and ω_c in equation (19) are the reference values for the robot's dynamic control input.

b) *Dynamic Control Law*

The objective of the dynamic control is for the linear velocity v and angular velocity ω of the robot to follow the values v_c and ω_c computed in equation (19). According to the synergetic control theory, the author selects the first technological invariants corresponding to the control objectives:

$$v = v_c;\ \ \omega = \omega_c \tag{21}$$

To implement cooperative control according to the synthesis process based on synergetic control theory, it is necessary to decompose the dynamic system (9) when there are no disturbances $d_{1,2} = 0$ and consider the set of objectives (21). To achieve this, it introduce the first macroscopic variable:

$$\psi_v = v - v_c;\ \ \psi_\omega = \omega - \omega_c; \tag{22}$$

The system of macroscopic variables (22), according to the technical characteristics, must satisfy the solution $\psi_v = 0,\ \psi_\omega = 0$ of functional equations:

$$\begin{cases} T_v\dot{\psi}_v + \psi_v = 0 \\ T_\omega\dot{\psi}_\omega + \psi_\omega = 0\end{cases} \tag{23}$$

Where $T_v > 0,\ T_\omega > 0$ ensure the conditions for the asymptotic stability of the system's motion. Substituting (22) and the system of equations (9) into the system of equations (23), we get:

$$\begin{cases} \left(m + \dfrac{2I_w}{R^2}\right)^{-1}\left(\dfrac{1}{R}\tau_v + m_c d\omega^2\right) = -\dfrac{1}{T_v}\psi_v + \dot{v}_c \\ \left(I + \dfrac{2l^2 I_w}{R^2}\right)^{-1}\left(\dfrac{l}{R}\tau_\omega - m_c d\omega^2\right) = -\dfrac{1}{T_\omega}\psi_\omega + \dot{v}_\omega\end{cases} \tag{24}$$

From the system of equations (24), we can derive the control laws τ_v, τ_ω in the form:

$$\begin{cases} \tau_v = \left(mR + \dfrac{2I_w}{R}\right)\left(-\dfrac{1}{T_v}\psi_v + \dot{v}_c\right) - Rm_c d\omega^2 \\ \tau_\omega = \left(\dfrac{R}{l}I + \dfrac{2lI_w}{R}\right)\left(-\dfrac{1}{T_\omega}\psi_\omega + \dot{v}_\omega\right) + \dfrac{R}{l}m_c dv\omega \end{cases} \quad (25)$$

IV. SIMULATING RESULTS

The following simulation experiments are conducted to simulate the robot tracking the desired reference trajectory over a time period of $t = 100\ (s)$ with a time step $dt = 0.005(s)$. The constants for the kinematic controller are $k_x = 2.8$, $k_y = 0.1$ and $k_\theta = 0.1$, while the constants for the synergetic controller for the dynamic system are $T_v = 0.1$ and $T_w = 0.1$. The robot model parameters are taken as follows from the study [14]: $m = 27(kg)$; $m_\omega = 0.5(kg)$; $I_c = 0.732(kgm^2)$, $I_\omega = 0.0025(kgm^2)$, $I_m = 0.0012(kgm^2)$, $d = 0.05(m)$, $R = 0.0975(m)$, $l = 0.164(m)$.

To demonstrate the quality of the proposed control law, simulation experiments are conducted with different trajectory tracking shapes, specifically a square trajectory and a figure-eight trajectory. Additionally, the simulation includes the controller to make the robot follow the desired reference trajectory while experiencing disturbances in the form of moment and position noise. For the moment disturbance, the disturbances d_v and d_ω are given by $d_v = -1.0 + random(-2{,}2)$, $d_\omega = 1.0 + random(-2{,}2)$. Simultaneously, position disturbances are also introduced during the tracking process. After every 8 seconds, the robot is made to slide in the x and y directions by a distance of 0.5 meters, with the first disturbance occurring at the 4th second.

A. Scenario 1

The trajectory to be followed is a square trajectory, represented by the following equation:

$$\begin{cases} x_d = \mathrm{a}|sin(wt)|\,sin(wt) \\ y_d = \mathrm{a}|cos(wt)|\,cos(wt) \end{cases} \quad (26)$$

Where $a = 10\ (m)$, $w = 0.2$. The robot's starting position is at the point $(0{,}0{,}0)$. The simulation results of the trajectory tracking are shown in Fig. 4 and Fig. 5. From Fig. 4, we can see that the robot can track the desired square trajectory well. Despite disturbances at moments when the robot slips off the desired trajectory, the results remain stable and converge back to the desired trajectory. As shown in Fig. 5, the errors in the x-axis, y-axis, and θ-angle are relatively small, even when slipping occurs and at the sharp corners of the trajectory.

B. Scenario 2

The trajectory to be followed is a figure-eight trajectory, represented by the following equation:

$$\begin{cases} x_d = \mathrm{a}\,sin(wt) \\ y_d = \mathrm{a}\,sin(wt)\,cos(wt) \end{cases} \quad (27)$$

Where $a = 10\ (m)$, $w = 0.2$. The robot's starting position is at the point $(0, a/5{,}0)$. The simulation results of the trajectory tracking are shown in Fig. 6 and Fig. 7. From Fig. 6, we can see that the robot can track the desired figure-eight trajectory well, with the system response showing minimal overshoot. Even when disturbances occur at moments when the robot slips off the desired trajectory, the results remain stable and converge back to the desired trajectory, though the angular error is larger compared to the square trajectory. As shown in Fig. 7, the errors in the x-axis and y-axis are relatively small even when the robot slips off the trajectory. Input disturbances with the proposed values have almost no impact on the quality of the robot's trajectory tracking in both cases. This confirms the effectiveness of the proposed control law.

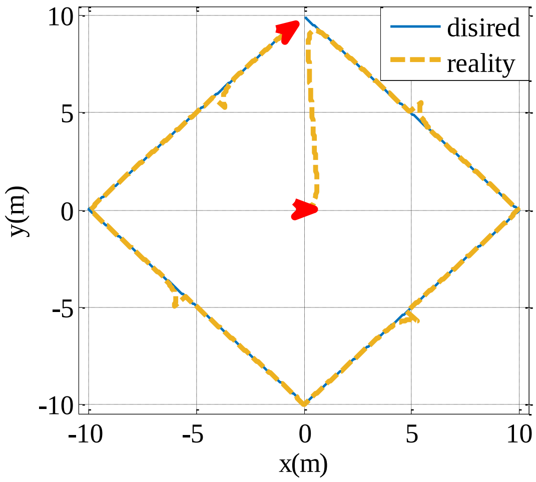

Fig. 4. Mobile robot trajectory with the starting point (0,0,0)

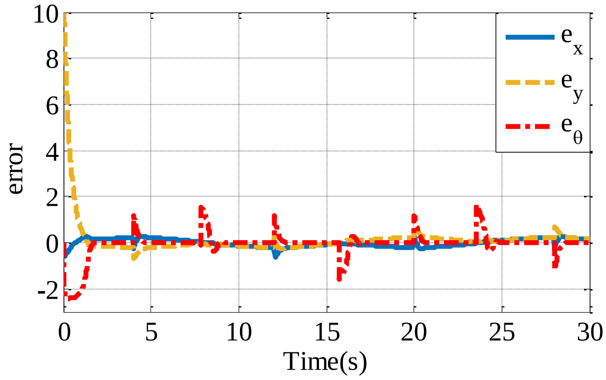

Fig. 5. Error in the three states x, y, θ of the mobile robot with the starting point $(0, 0, 0)$

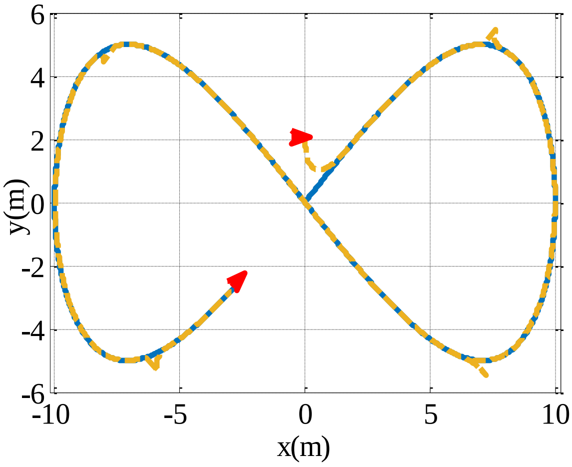

Fig. 6. Mobile robot trajectory with the starting point (0,a/5,0)

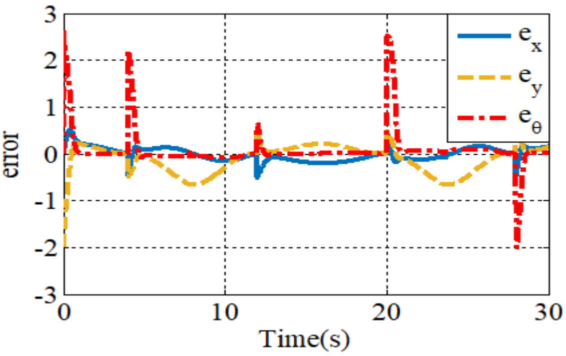

Fig. 7. Error in the three states x, y, θ of the mobile robot with the starting point $(0, a/5, 0)$

V. CONCLUSION

In this paper, a method for synthesizing a control law for a mobile robot to follow a desired trajectory is presented. The control law for the robot is designed with two control loops: the kinematic control law is built based on the Lyapunov function, and the dynamic control law is synthesized based on the hybrid control theory. Simulation results with different scenarios under disturbance conditions demonstrate the effectiveness of the proposed control law in trajectory tracking and its resilience to disturbances. Furthermore, it is observed that the selection of parameters in the control law significantly affects the control quality, and the global stability of the control system needs to be proven. Therefore, in future studies, the authors will present methods to address these drawbacks.

REFERENCES

[1] C. Samson and K. Ait-Abderrahim, "Feedback control of a nonholonomic wheeled cart in Cartesian space," *Proceedings. 1991 IEEE International Conference on Robotics and Automation*, Sacramento, vol. 2, pp. 1136-1141, 1991, https://doi.org/10.1109/ROBOT.1991.131748.

[2] M. Aicardi, G. Casalino, A. Bicchi and A. Balestrino, "Closed loop steering of unicycle like vehicles via Lyapunov techniques," *IEEE Robotics & Automation Magazine*, vol. 2, no. 1, pp. 27-35, 1995, https://doi.org/10.1109/100.388294.

[3] Byung Moon Kim and P. Tsiotras, "Controllers for unicycle-type wheeled robots: Theoretical results and experimental validation," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 3, pp. 294-307, 2002, https://doi.org/10.1109/TRA.2002.1019460.

[4] G. Zidani, S. Drid, L. Chrifi-Alaoui, A. Benmakhlouf and S. Chaouch, "Backstepping controller for a wheeled mobile robot," *2015 4th International Conference on Systems and Control (ICSC)*, pp. 443-448, 2015, https://doi.org/10.1109/ICoSC.2015.7153286.

[5] I. Hassania, C. Rekik, "Backstepping Controller for Mobile Robot in Presence of Disturbances and Uncertainties," *International Journal of Robotics and Control Systems*, vol. 3, no. 4, pp. 934-954, 2023, https://doi.org/10.31763/ijrcs.v3i4.1110.

[6] T. P. Nascimento, C. E. T. Dórea, and L. M. G. Gonçalves, "Nonlinear model predictive control for trajectory tracking of nonholonomic mobile robots: A modified approach," *International Journal of Advanced Robotic Systems*, vol. 15, no. 1, 2018, https://doi.org/10.1177/1729881418760461.

[7] S. Bouzoualegh, E.-H. Guechi, and R. Kelaiaia, "Model predictive control of a differentialdrive mobile robot," *Acta Universitatis Sapientiae, Electrical and Mechanical Engineering*, vol. 10, no. 1, pp. 20-41, 2018, https://doi.org/10.2478/auseme-2018-0002.

[8] J. Yang and J. Kim, "Sliding mode control for trajectory tracking of nonholonomic wheeled mobile robots," *IEEE Transactions on Robotics and Automation*, vol. 15, no. 3, pp. 578-587, 1999, https://doi.org/10.1109/70.768190.

[9] Felipe N. Martins, Wanderley C. Celeste, Ricardo Carelli, Mário Sarcinelli-Filho, Teodiano F. Bastos-Filho, "An adaptive dynamic controller for autonomous mobile robot trajectory tracking Author links open overlay panel," *Control Engineering Practice*, vol 16, no 11, pp. 1354-1363, 2008, https://doi.org/10.1016/j.conengprac.2008.03.004.

[10] R. Barzamini, A. R. Yazdizadeh and A. H. Rahmani, "A New Adaptive Tracking Control For Wheeled Mobile Robot," *2006 IEEE Conference on Robotics, Automation and Mechatronics*, pp. 1-6, 2006, https://doi.org/10.1109/RAMECH.2006.252750.

[11] H. Omrane, M. S. Masmoudi, M. Masmoudi, "Fuzzy Logic Based Control for Autonomous Mobile Robot Navigation," *Computational Intelligence and Neuroscience*, vol. 2016, no. 1, pp. 1-10, 2016, https://doi.org/10.1155/2016/9548482.

[12] F. Cupertino, V. Giordano, D. Naso and L. Delfine, "Fuzzy control of a mobile robot," *IEEE Robotics & Automation Magazine*, vol. 13, no. 4, pp. 74-81, 2006, https://doi.org/10.1109/MRA.2006.250563.

[13] M. Akbarzadeh-T, "Agencies of Intelligence: From the Macro to the Nano," *Control and Systems Engineering, Springer*, pp. 289-311, 2015, https://doi.org/10.1007/978-3-319-14636-2_15.

[14] N. X. Chiem, N. D. Anh, A. D. Lukianov, P. D. Tung, H. D. Long, and N. D. Linh, "Design real-time embedded optimal PD fuzzy controller by PSO algorithm for autonomous vehicle mounted camera," *AIP Conference Proceedings*, vol. 2188, no. 1, p. 030008, 2019, https://doi.org/10.1063/1.5138401.

[15] A. A. Kolesnikov, "Introduction of synergetic control," *2014 American Control Conference*, pp. 3013-3016, 2014, https://doi.org/10.1109/ACC.2014.6859397.

[16] A. A. Kolesnikov, "Introduction of synergetic control," *2014 American Control Conference*, pp. 3013-3016, 2014, https://doi.org/10.1109/ACC.2014.6859397.

[17] C. S. Nguyen, C. V. Tran, H. N. Phan, "Control Law Synthesis for Flexible Joint Manipulator Based on Synergetic Control Theory," *Mechatronics, Automation, Control*, vol. 24, no. 8, pp. 395-402, 2023, https://doi.org/10.17587/mau.24.395-402.

[18] Nguyen Xuan Chiem, Le Tran Thang, "Synthesis of an Orbit Tracking Controller for a 2DOF Helicopter based on Sequential Manifolds with Stabilization Time in the Presence of Disturbances," *Engineering, Technology & Applied Science Research*, vol. 14, no. 4, pp. 15083-15089, 2024, https://doi.org/10.48084/etasr.7512.

[19] N. X. Chiem, B. X. Hai, T. C. Phan, "Synthesis of Adaptive Sliding Mode Control for Twin Rotor MIMO System with Mass Uncertainty based on Synergetic Control Theory," *International Journal of Robotics and Control Systems*, vol. 4, no. 1, pp. 174-187, 2024, https://doi.org/10.31763/ijrcs.v4i1.1307.