

# Hybrid Fuzzy-Fractional Order PID Control for Industrial Hydraulic Actuators System: Experimental Study

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## ABSTRACT

Due to its fast response and high reliability, the Industrial Hydraulic Actuator (IHA) system is widely preferred in modern industrial applications. The system integrates both electrical and hydraulic components; however, its inherent nonlinearities and significant model uncertainties make the design of an accurate trajectory-tracking controller a considerable challenge. This study proposes a Hybrid Fuzzy Fractional-Order Proportional-Integral-Derivative (HF-FOPID) controller to enhance the trajectory-tracking performance of an actual IHA system. An experimental workbench was established to collect real-time data, and a conventional FOPID controller was first developed with its parameters optimized using Particle Swarm Optimization (PSO). The HF-FOPID controller incorporates fuzzy logic control framework, providing improved adaptability and effectiveness. Controller performance was validated using two types of trajectories and assessed through error metrics, including Mean Squared Error (MSE) and Root Mean Squared Error (RMSE) where lower values indicate higher control accuracy. Experimental results demonstrate that the HF-FOPID controller outperforms the FOPID controller, achieving improvements in MSE and RMSE of 97.17% and 83.17%, respectively for point-to-point (PTP) tracking, and 57.48% and 34.79% for multi-sinusoidal tracking. These findings confirm that the HF-FOPID controller delivers superior high-precision trajectory tracking compared to conventional linear controllers.

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## 1. Introduction

Industrial Hydraulic Actuator (IHA) systems are recognized for their high precision, stability, power efficiency, and environmental benefits. These advantages have driven their wide application in aerospace, robotics, and vehicle suspension systems [1]-[5]. However, recent advancements in IHA design, such as ultra-high power, lightweight structures, and rapid response, have also introduced challenges. Issues like abnormal vibrations and system damage have become more frequent [6]-[8]. Such problems arise mainly from the system's inherent nonlinearities and significant model uncertainties [9]-[12], including parameter variations, unmodeled dynamics, and external disturbances. These uncertainties can reduce the stability and effectiveness of controllers designed using nominal models. To address these challenges, researchers have focused on two main directions. First, accurate mathematical models have been developed to capture nonlinear characteristics and improve predictive capability [13], [14]. Second, advanced control algorithms have been proposed, including disturbance observer-based methods, adaptive strategies, and sliding mode control, which improve tracking and disturbance rejection under uncertain conditions [15], [16]. In addition, IHAs are increasingly used in emerging applications, such as energy-efficient vehicle powertrains [6], renewable energy systems [9], and fault-tolerant suspensions [11]. These applications further highlight the need for robust modeling and control solutions to ensure reliable performance.

Several control strategies have been applied to IHA systems, including linear [17]-[23], nonlinear [24]-[33], intelligent [34]-[42], and hybrid [43]-[51] approaches. These methods aim to analyze system behavior under varying conditions and generate appropriate control inputs to ensure stable and reliable operation. Building on these efforts, this study proposes a hybrid controller that integrates linear and intelligent control techniques. The developed controller is designed to achieve accurate trajectory tracking while maintaining consistent performance under diverse environmental conditions. In doing so, the research enhances the adaptability and reliability of IHA systems in practical applications.

The linear control approach is widely applied in IHA systems because of its simplicity, with PID control being one of the most commonly used techniques for trajectory tracking [17]-[19]. For instance, Feng et al. [17] introduced an improved PSO-PID controller for electro-hydraulic systems, demonstrating enhanced trajectory control performance. Similarly, Yang et al. [18] analyzed PID event-trigger strategies, further confirming its industrial practicality. Despite these strengths, the effectiveness of PID control declines in highly nonlinear systems, often leading to reduced accuracy and adaptability [20]. To address these limitations, the Fractional Order PID (FOPID) controller has been proposed, which extends the conventional PID by incorporating  $\lambda$  (integral order) and  $\mu$  (derivative order) terms [21]. Studies such as those by Atsari and Halim [21] and Nataraj and Subramaniam [22] highlight its capability to achieve improved dynamic response and control flexibility. However, the increased flexibility also complicates parameter tuning, leading many studies to employ optimization techniques such as metaheuristic algorithms for achieving optimal controller performance [23].

Linear control methods often struggle to accurately regulate nonlinear systems, primarily due to difficulties in parameter tuning and their inability to ensure stability under varying operating conditions [24]. These drawbacks can cause significant performance degradation and even instability in the presence of modeling uncertainties [25]-[27]. To overcome such challenges, the backstepping control strategy has been proposed, offering both system stabilization and reliable trajectory tracking. For instance, Enyan et al. [25] applied a Lyapunov-based robust integral backstepping controller to an electro-hydraulic servo system, while Eny et al. [26] investigated its velocity control and chattering characteristics. Further improvements have been demonstrated through differentiator-based and model-optimized designs [27]-[29]. In addition, Sliding Mode Control (SMC), a nonlinear variable structure control approach, has been widely adopted to address the inherent nonlinearities of IHA systems [30], [31]. Despite their effectiveness, both backstepping and SMC remain model-based approaches, requiring accurate system models and involving complex controller design [32], [33].

Moreover, intelligent control leverages artificial intelligence techniques to design adaptive controllers capable of handling dynamic environments and improving system performance. Among the widely used methods are Artificial Neural Networks (ANN) [34], [36] and Fuzzy Logic Control (FLC) [37]-[42], both of which demonstrate strong capabilities in approximating continuous nonlinear functions. These approaches are particularly effective for managing the challenges of uncertain and highly nonlinear systems [36]. Despite their advantages, FLC heavily relies on expert knowledge for rule definition and tuning (Jiang et al. [37]; Van et al. [38]), while ANN requires extensive training data to achieve reliable trajectory tracking and robust performance (Chanbua and Pinsopon [40]; Song et al. [41]).

Furthermore, hybrid control strategies have been extensively explored to improve robustness, adaptability, and efficiency in electro-hydraulic systems. For instance, Fadel [43] developed a PSO-optimized PID–SMC controller for electro-hydraulic actuators, achieving improved robustness at the expense of higher design complexity. Similarly, Baharuddin and Basri [44] applied fuzzy logic for self-tuning PID in quadcopters, while Wu et al. [45] demonstrated the ability of fuzzy–PID control to handle nonlinearities in an electro-hydraulic thrust system. Further advancements include the integration of fuzzy rules into sliding mode control, as reported by Feng et al. [46], Tho et al. [47], and Ghani et al. [48], which enhanced adaptability and tracking accuracy in actuator applications. Building on this, Ali et al. [49] introduced an enhanced hybrid fuzzy-PID controller for precise trajectory tracking in electro-hydraulic actuators, demonstrating significant improvements in accuracy and robustness. These contributions confirm the potential of hybrid control approaches, though most remain limited to PID-SMC or fuzzy-PID/SMC structures. As observed in linear control approaches, FOPID provides improved performance compared to conventional PID [50], [51]. However, its practical application remains constrained by implementation complexity and limited adaptability under nonlinear and uncertain conditions.

Optimal performance relies on effective parameter tuning, as manual methods are often inadequate. Advanced optimization techniques, such as Snake Optimization (SO) [52], Genetic Algorithm (GA) [53], and Grey Wolf Optimizer (GWO) [54], are commonly employed. SO, balances exploration and exploitation through dual competitive and cooperative phases while GA evolves a population of candidate solutions via selection, crossover, and mutation. GWO leverages social hierarchy and cooperative hunting strategies to efficiently find optimal solutions. However, these methods can be computationally intensive, sensitive to initial conditions, and may require careful parameter tuning to avoid premature convergence. In contrast, Particle Swarm Optimization (PSO), inspired by the social behavior of bird flocking and fish schooling, provides a simple and efficient approach [55] for finding optimal design parameters and will be used in this study. The contribution of this paper is summarized as follows:

- Application of Particle Swarm Optimization (PSO) for tuning the parameters of the FOPID controller, enabling efficient and adaptive parameter selection for IHA systems.
- Development of a parallel hybrid fuzzy–FOPID controller that integrates fuzzy logic with the optimized FOPID structure to enhance trajectory tracking accuracy under different trajectories.

## 2. Methodology

### 2.1. Research Flowchart

This section presents the research process conducted on the IHA system. The research focuses on designing controllers that can be used in real-time experiments of IHA systems. It is also carried out using various types of inputs and performance tracking indices as the result.

Fig. 1 represents flowchart of this research. This research started with the experimental workbench of IHA system which was provided with all the equipment. Next, based on the actual system, the system model for the control settings could be identified. The system identification model was established in this work, as reported in [56]. Furthermore, the control design started with FOPID

control using Particle Swarm Optimization (PSO) tuning method and after that it went to Fuzzy-FOPID control design. After that, all two controllers were validated and compared using various tracking responses in real-time experiments. The tracking performance was observed based on the error analysis for the results.

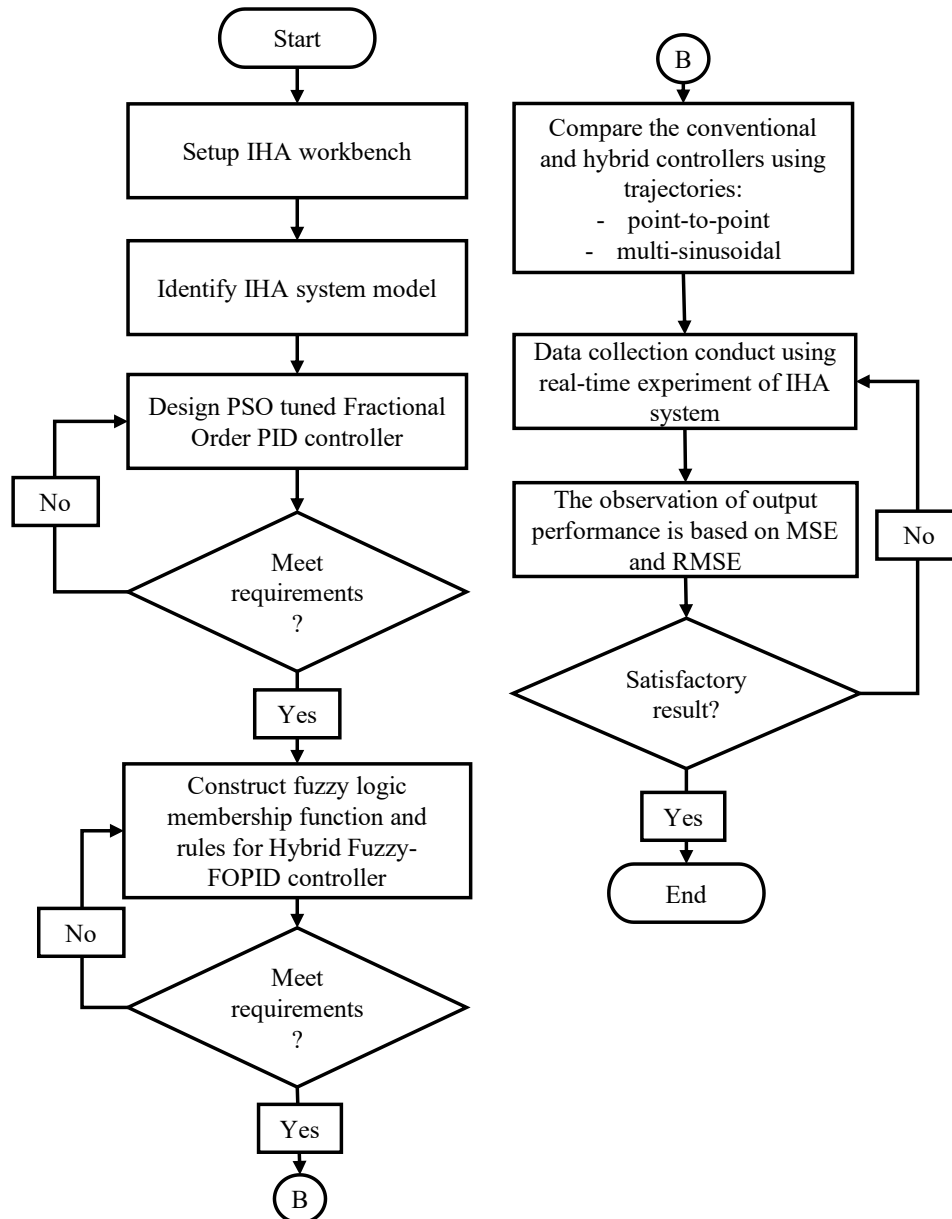


Fig. 1. Flowchart of the research

## 2.2. IHA System Workbench

Fig. 2 shows the real-time experiment workbench of the IHA system. The computer is installed by MATLAB and SIMULINIK version R2021a software that are used for simulation and experimental work of modelling and controller design. The proportional directional valve manufactured by Bosch Rexroth with the model number 4WWREE6E08-2X/G24K31/A1V is used to move the hydraulic actuator. The proportional directional valve has a maximum flow rate of 80L/min, maximum pressure of 31.5 MPa and a command value of 10-volts. The double-acting cylinder of hydraulic actuator is also considered in this research. The hydraulic cylinder features a 40 mm bore, a 25 mm rod and a stroke of 200 mm. The hydraulic power pack with 60-liter tank, 16.6 liter per minute of flow and 5 MPa maximum supply pressure are used in this research. PCIe-6321

card was manufactured by National Instruments considered in this research as Data Acquisition System (DAQ). The number of analogue output channels for PCIe-6321 card is 2 with  $\pm 10$  and the sample rate are 250000 samples per second.

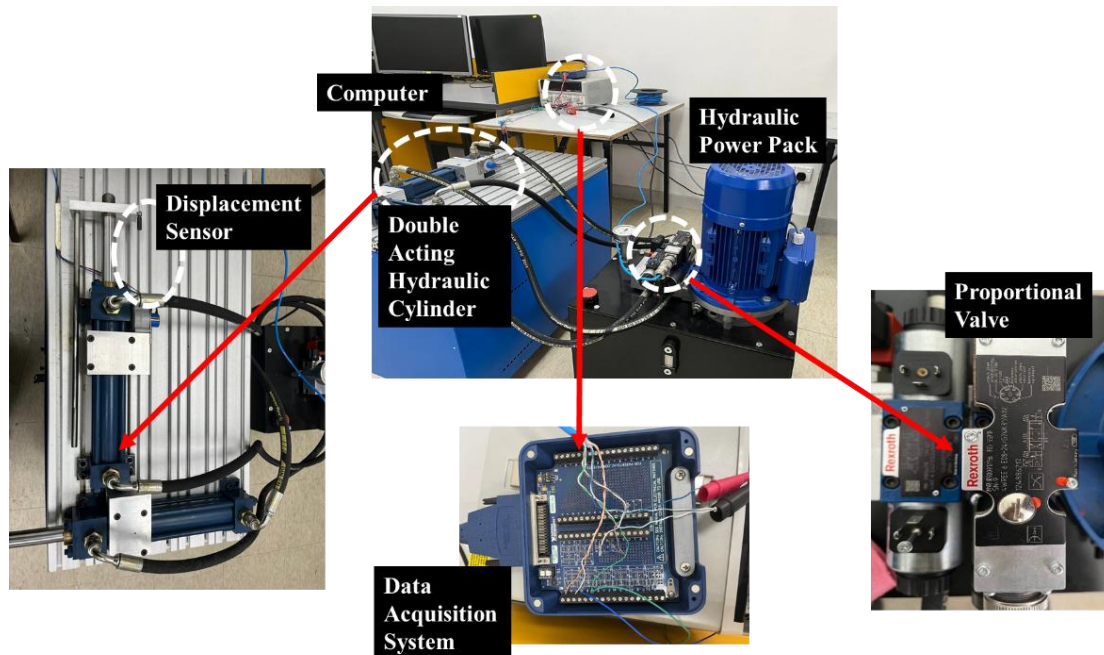


Fig. 2. IHA system experimental workbench

### 2.3. FOPID Control Design with PSO Approach

Unlike the conventional PID controller, which is defined by three main parameters: proportional  $K_p$ , integral  $K_i$  and derivative  $K_d$ , the FOPID controller extends this structure by introducing two additional parameters: the fractional integral order,  $\gamma$  and fractional derivative order,  $\mu$  [57]. These additional parameters modify the behavior of the integral and derivative terms, allowing for more flexible and precise control. The transfer function of a standard PID controller is typically expressed as:

$$G(s)_{PID} = K_p + K_i \frac{1}{s} + K_d s \quad (1)$$

Here  $K_p$  represents the proportional gain,  $K_i$  denotes the integral gain and  $K_d$  indicates derivative gain. By incorporating the additional fractional orders, the transfer function of FOPID controller is formulated as follows:

$$G(s)_{FOPID} = K_p + K_i s^{-\lambda} + K_d s^{\mu} \quad (2)$$

The fractional order of  $\gamma$  and  $\mu$  are not restricted to integer values. Previous studies have shown that incorporating these fractional integral and derivative orders to form what is known as the FOPID can enhance the performance of conventional PID controllers [58]. However, from a computational standpoint, the addition of these extra parameters increases the complexity of the tuning process, resulting in longer computation times for parameter identification. Since the linear identification is using discrete-time model, the FOPID controller need to be converted to discrete-time structure. The transformation from continuous to discrete is using Bilinear/Tustin Transform [59]. The Tustin transformation maps the continuous to discrete is as follows:

$$s = \frac{2}{T_s} \left( \frac{z-1}{z+1} \right) \quad (3)$$

The sampling time,  $T_s$  is 0.01 for the transformation. the transformation only occurs on the integral and also the derivative gain that has a s-domain change to represent the gain, proportional remains with no changes to be made. Equation (3) then is inserted to the s-domain FOPID control transfer function which resulting the new transfer function of FOPID controller. Characteristics of fractional order PID shown in Fig. 3.

$$G(z)_{FOPID} = K_p + K_i \left( \frac{2}{T_s} \frac{z-1}{z+1} \right)^{-\lambda} + K_d \left( \frac{2}{T_s} \frac{z-1}{z+1} \right)^{\mu} \quad (4)$$

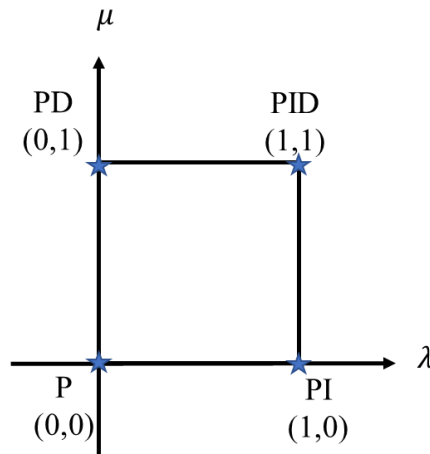


Fig. 3. Characteristics of fractional order PID

The FOPID controller was subsequently simulated using the PSO algorithm to determine the optimal parameter values, as illustrated in the block diagram shown in Fig. 4. The reference trajectory signal is denoted by  $r$  while the piston trajectory signal is denoted by  $x_p$ . The system tracking error,  $e$  is defined by the equation in (5).

$$e = r - x_p \quad (5)$$

PSO is one of the established optimal tuning methods to give optimal values of the design parameters/variables and it was designed by mimicking the swarm social behavior of bird flocking and fish schooling in order to search and move to the food with certain speed and position [60]. It has been used to solve various optimization problems because it is a well-established method and it is easy to use. This particle position in PSO for this research can be modeled as equation (6).

$$x^i = [K_p, K_i, K_d, \lambda, \mu] \quad (6)$$

$x^i$  is the particle position, while  $K_p, K_i, K_d, \lambda, \mu$  are the proportional, integral, derivative, lamda and mu, respectively values for FOPID controller gain. The process started with a random number of particles and it is performed by using equation (7).

$$x^i = x_{min} + rand(x_{max} - x_{min}) \quad (7)$$

The parameter  $x_{max}$  and  $x_{min}$  represent the boundary constraints of the search space which is 10 and 0.2, respectively. During each iteration, particles explore the search space to identify their respective local best position,  $P_{BEST}$  and subsequently determine the global best position,  $G_{BEST}$  to approach an optimal solution. Each particle adapts its trajectory by balancing exploitation of its previous successes and exploration guided by the most successful agent in the swarm. Updates to  $P_{BEST}$  and  $G_{BEST}$  occur when a particle achieves a fitness value superior to the current best values, provided it remains within the systems constrains. The update velocity and position of each particle are computed according to equations (8) and (9) accordingly.

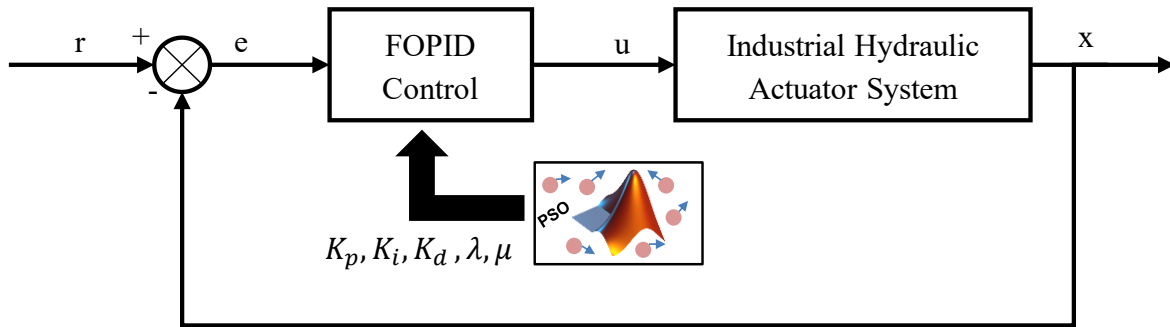


Fig. 4. PSO tuned FOPID schematic diagram

$$v^{i+1} = wv^i + c_1r_1 (P_{BEST} - x^i) + c_2r_2 (G_{BEST} - x^i) \quad (8)$$

$$x^{i+1} = x^i + v^{i+1} \quad (9)$$

The variables  $r_1$  and  $r_2$  represent the randomly generated values within the interval  $[0,1]$  while  $c_1$  and  $c_2$  denote the cognitive and social components, respectively. The particle size was configured to 20, with a maximum of 100 iterations, and both the cognitive and social coefficients were set to 2. The performance of each particle was assessed using a fitness function  $J$ , which is based on the integral of absolute error (IAE). The corresponding equation for the fitness function is presented in (10). Fig. 5 presents the overall process of the Particle Swarm Optimization (PSO) algorithm used to obtain the optimal gain parameters of the FOPID controller. The process starts with initializing particles in random positions and velocities, followed by evaluating each particle through the Simulink model of the FOPID system. The fitness value  $J$  is calculated, the personal best and global best are updated accordingly. Particle positions and velocities are iteratively adjusted using equations (8) and (9) until the maximum iteration limit is reached [61]. Finally, the optimal parameters are obtained and substituted into equation (4) for implementation. The optimal parameter of FOPID is  $k_p=0.5362$ ,  $k_i=0.7207$ ,  $k_d=0.0367$ ,  $\lambda=0.0456$ ,  $\mu=0.1679$ . The parameter was also utilized in the proposed hybrid fuzzy-FOPID controller.

$$J = \int_0^T |x_d(t) - x_r(t)| dt \quad (10)$$

#### 2.4. Hybrid Fuzzy-FOPID Control Design

IHA systems exhibit highly nonlinear and complex dynamic characteristics, which pose significant challenges to the implementation of conventional linear control techniques. To address this issue, fuzzy logic is employed due to its use of linguistic variables and flexible rule-based frameworks that allow effective modeling and control of such systems. The fuzzy rule base can be constructed using expert knowledge and further refined to enhance overall system performance. In this study, a hybrid control strategy is proposed by integrating fuzzy logic with a FOPID controller. This hybrid configuration leverages the strengths of both approaches, with the fuzzy rules applied to complement the FOPID structure. The Mamdani-type fuzzy inference system is adopted for rule development, implemented using the MATLAB Fuzzy Logic Toolbox (as shown in Fig. 6). Fig. 7 and Fig. 8 depict the membership functions for the input error, the rate of change of error, and the output voltage, respectively. The corresponding fuzzy rule base, presented in Table 1, consists of nine rules derived from three linguistic terms for both the error and its rate of change, resulting in nine rule combinations mapped to five linguistic output voltage levels. The rule set was developed using a trial-and-error method to achieve desirable control performance.

The fuzzy inference system was developed using two input variables, the position error and the derivative of error with membership functions defined over the ranges  $[-0.4,0.4]$  and  $[-100,100]$  respectively. Each input is represented by three linguistic terms: Negative (N), Zero (Z) and Positive

(P). The output corresponds to the auxiliary control voltage applied to valve, ranging from  $[-3,3]$  and is defined by five linguistic terms: Big Negative (BN), Negative (N), Zero (Z), Positive (P) and Big Positive (BP). This design establishes a mapping between the input conditions and the corrective voltage signal, as shown in the membership functions.

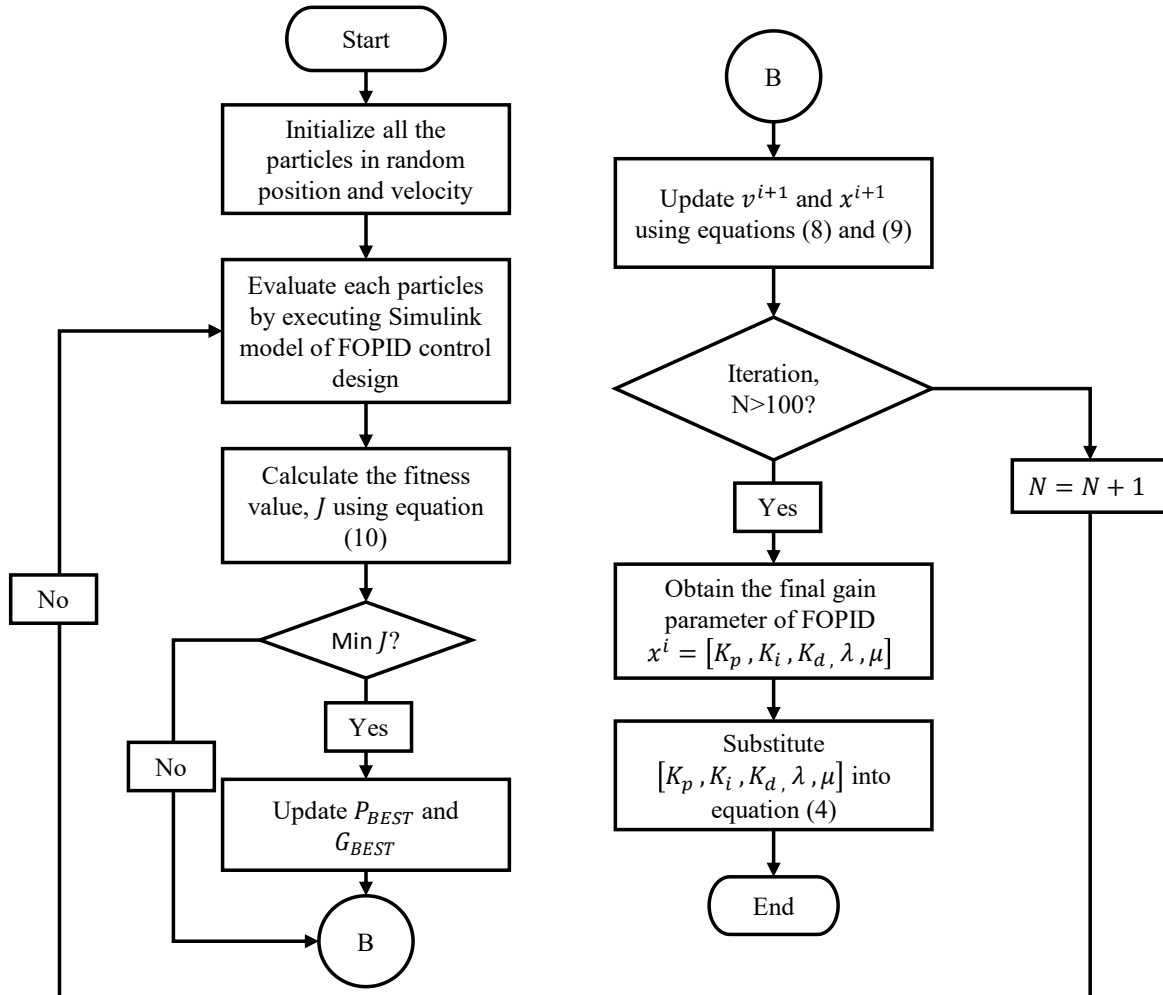


Fig. 5. PSO design process

Moreover, the constructed rule base is designed to capture intuitive control actions that resemble the decision-making process of a human operator. For example, when both the error and its derivative are negative (N, N), the actuator is diverging from the reference trajectory in the negative direction. To counter this condition, the fuzzy controller produces a Big Positive (BP) output voltage, providing a strong corrective signal to drive the actuator back toward the desired position. Conversely, when both the error and its derivative are close to zero (Z, Z), the output is set to Zero (Z), indicating that no additional correction is required since the FOPID controller alone can sustain the trajectory. Similarly, when the error and its derivative are both positive (P, P), the actuator tends to overshoot in the positive direction, and the fuzzy controller responds with a Big Positive (BP) output to mitigate the overshoot and stabilize the motion. For intermediate conditions, such as when the error is small but the derivative is non-zero, the controller issues moderate outputs (e.g., N or P), ensuring smoother adjustments without imposing abrupt changes on the valve input. In this way, the fuzzy controller acts as a supportive element by improving the trajectory tracking performance of the IHA system.

The block diagram of the proposed Fuzzy-FOPID controller is illustrated in Fig. 9. The overall structure of the controller is designed in a parallel configuration, where both fuzzy and FOPID controller operate simultaneously. In this configuration, each controller independently generates an

output voltage signal based on the system error and its rate of change. These output signals are then combined to form the final control signal that drives the actuator. The parallel structure enables the hybrid controller to leverage the strengths of both control strategies. The FOPID controller contributes precise linear control based on fractional calculus, while the fuzzy logic controller enhances adaptability and robustness by handling system nonlinearities and uncertainties. The combined control output allows for improved system performance compared to using either controller alone. Specifically, the additional contribution from the fuzzy controller enhances the conventional FOPID controller's ability to respond effectively [62]. As a result, the hybrid Fuzzy-FOPID controller demonstrates faster response and improved adaptability.

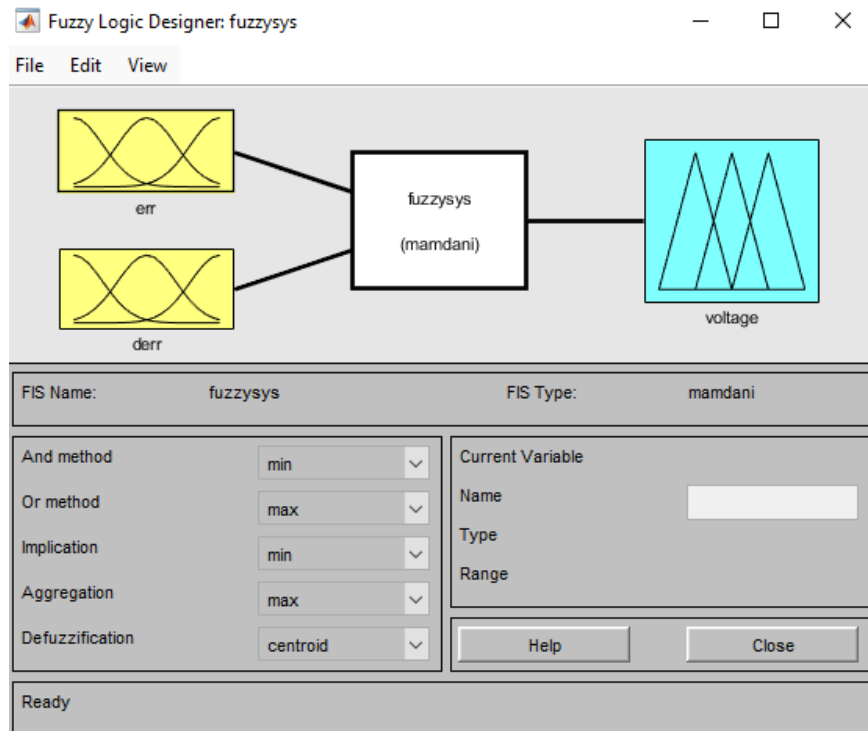


Fig. 6. Fuzzy logic toolbox

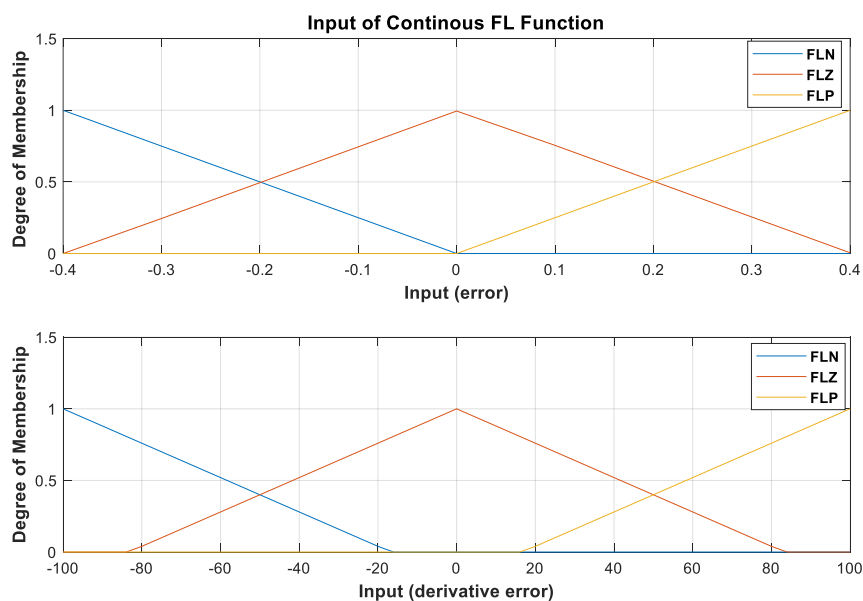


Fig. 7. Input membership function

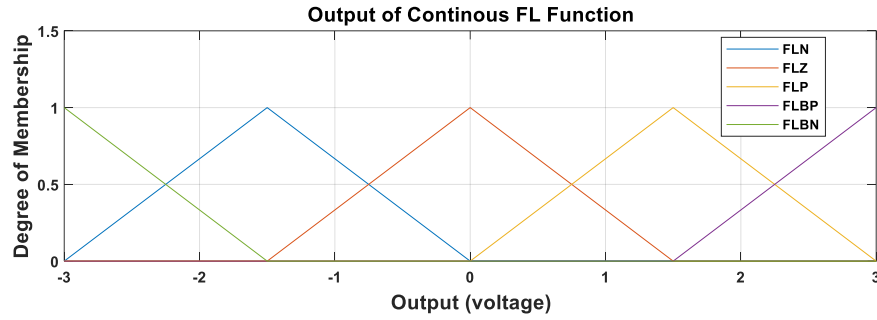


Fig. 8. Output membership function

Table 1. Fuzzy rules

		Error		
		N	Z	P
Rate Change of Error, de/dt	N	BP	N	N
	Z	BN	Z	BP
	P	P	P	BP

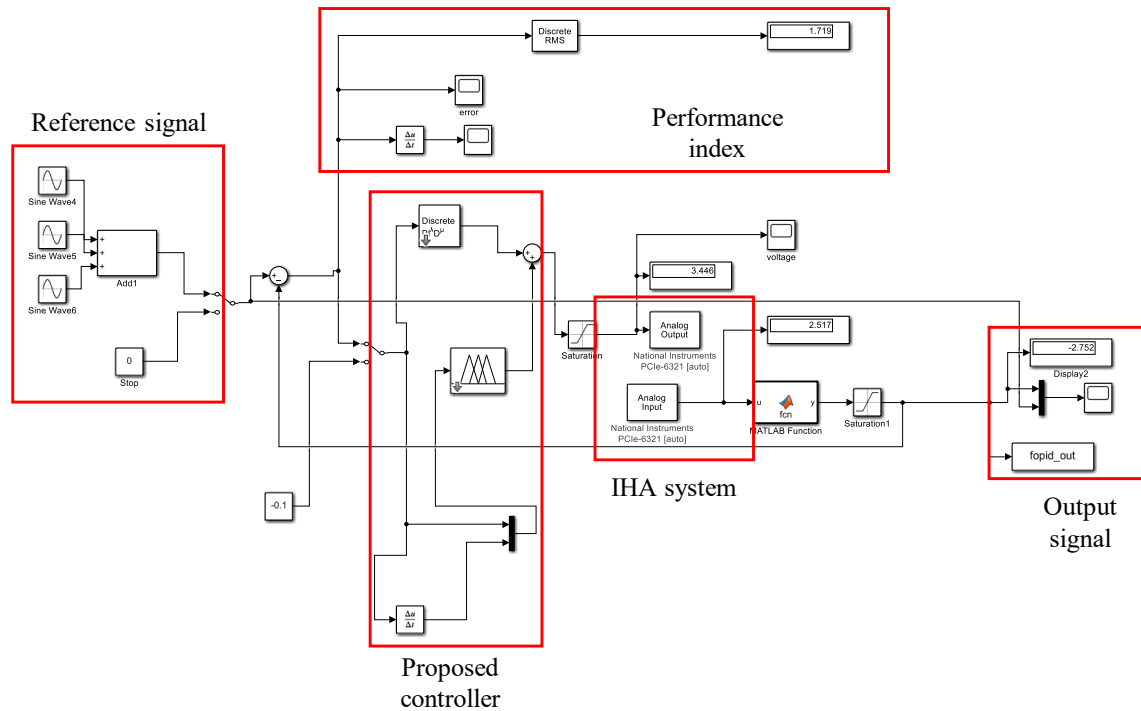


Fig. 9. Block diagram of hybrid Fuzzy-FOPID controller

2.5. Step Input, Point-to-Point and Multi-Sinusoidal Trajectory

The trajectory represents the intended path or motion that a hydraulic actuator is expected to follow within an IHA system, as illustrated in Fig. 10 and Fig. 11. A PTP trajectory refers to a specific control method that directs the actuator to move precisely from a defined starting position to a designated endpoint. This approach aims to minimize tracking errors and enhance the accuracy of actuator positioning. Besides that, multi-sinusoidal trajectories are among the most frequently utilized motion profiles. These involve the superposition of several sinusoidal signals, each with unique frequencies and amplitudes, resulting in a complex movement pattern. Such trajectories are widely applied in aerospace, agriculture and heavy industries. The generation and execution of these complex trajectory patterns in hydraulic systems can be effectively managed through suitable control strategies and signal generation techniques.

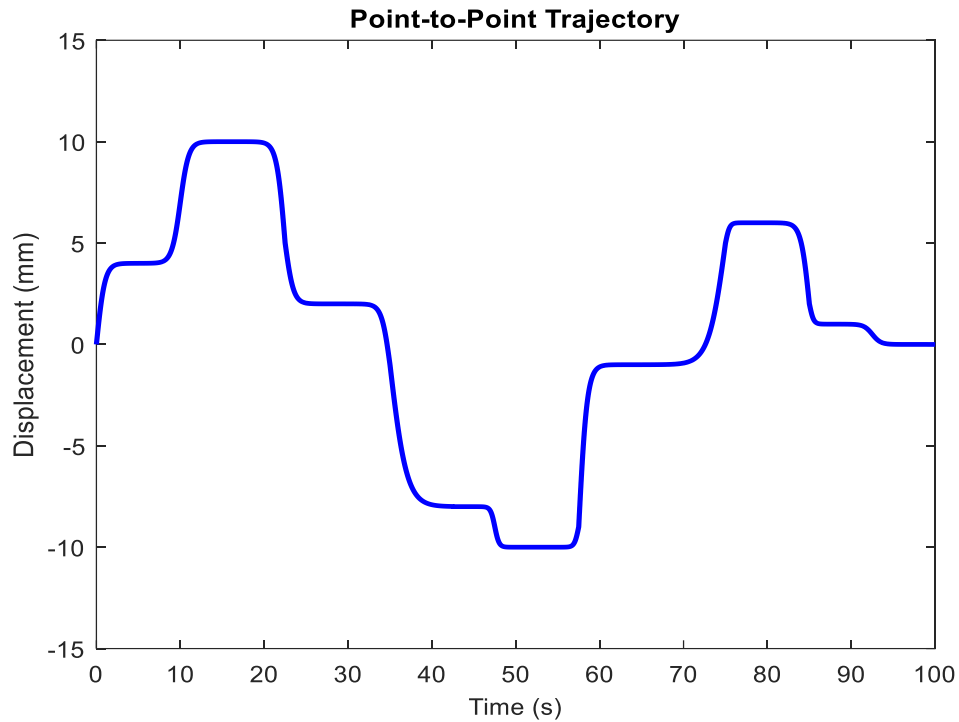


Fig. 10. Point-to-point trajectory

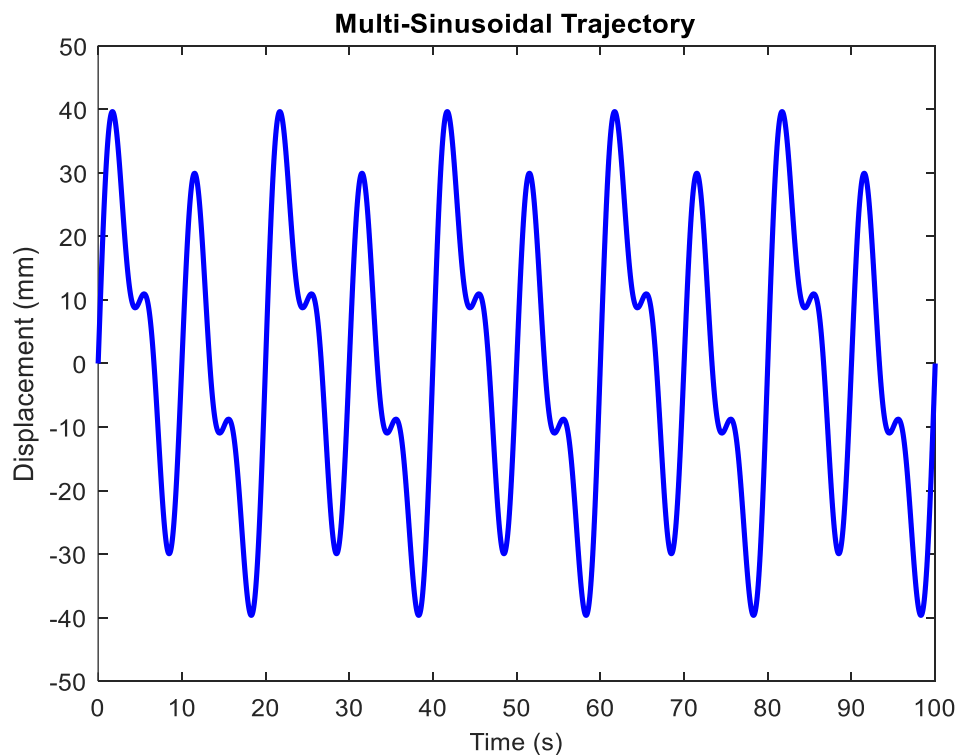


Fig. 11. Multi-sinusoidal trajectory

### 3. Results and Discussion

#### 3.1. Trajectory Tracking Response

Fig. 12 and Fig. 13 illustrate the trajectory tracking responses of both controllers. The results clearly indicate that the Fuzzy-FOPID controller delivers superior performance compared with the

conventional FOPID controller. The output of the Fuzzy-FOPID controller remains consistently closer to the reference trajectory, reflecting its ability to achieve higher tracking accuracy and enhanced stability. This improvement can be attributed to the integration of fuzzy logic, which provides adaptive adjustment of the control parameters and enables the system to more effectively minimize errors during operation. In contrast, the FOPID controller exhibits larger deviations from the reference signal, particularly during rapid changes in the trajectory. While the FOPID controller is capable of producing satisfactory results, its performance is comparatively limited in managing system nonlinearities and variations. The adaptive nature of the Fuzzy-FOPID structure allows it to respond more effectively to these challenges, thereby ensuring more precise and reliable trajectory tracking. To complement the graphical results, a quantitative error analysis was performed as summarized in Table 2. The error metrics further validate the superior performance of the Fuzzy-FOPID controller, demonstrating its ability to consistently reduce deviations. These outcomes confirm the effectiveness of the proposed controller in achieving accurate, stable, and reliable tracking performance.

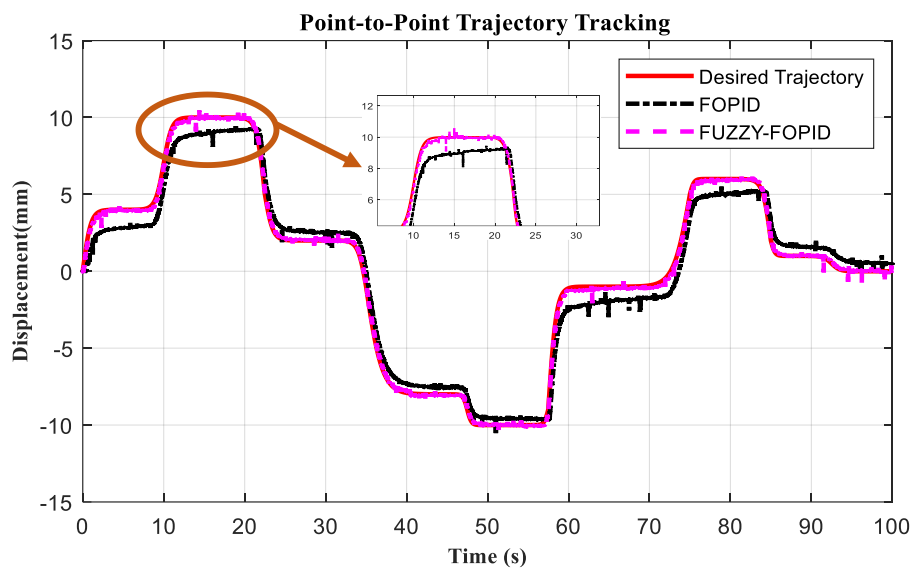


Fig. 12. Point-to-point trajectory tracking response for fuzzy logic controller

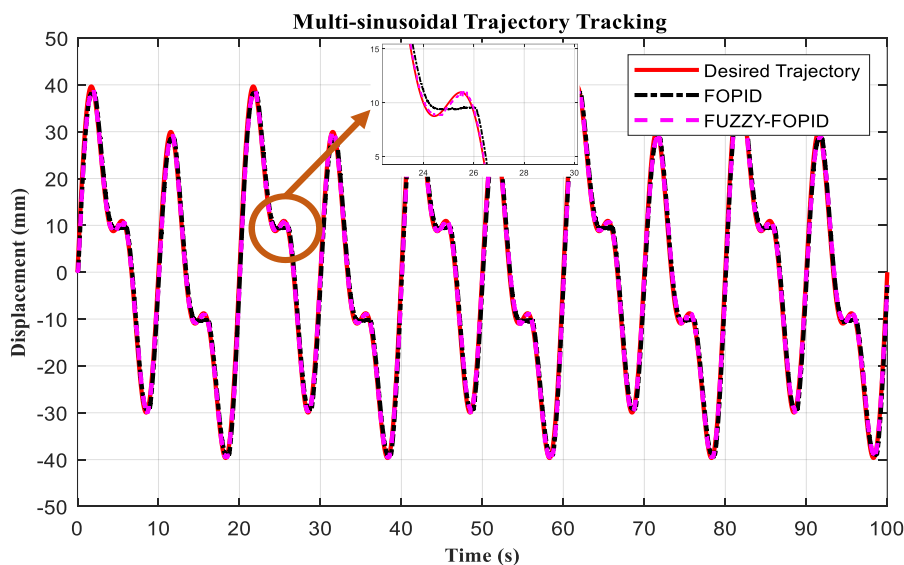


Fig. 13. Multi-sinusoidal trajectory tracking response for fuzzy logic controller

Table 2 presents the error analysis of trajectory tracking performance using MSE and RMSE. For the PTP trajectory, the Fuzzy-FOPID controller demonstrates substantial improvements compared

with the conventional FOPID controller. The MSE decreases from 0.8446 to 0.0239, corresponding to a reduction of 97.17%, while the RMSE is reduced from 0.9190 to 0.1547, representing an improvement of 83.17%. In absolute terms, these reductions indicate only minimal deviations between the output response and the reference signal. Such results confirm that the system achieves smoother transitions, more accurate positioning, and reduced fluctuations. From a practical perspective, this improvement enables precise motion control, which is particularly valuable in applications requiring high accuracy, such as robotic manipulators, aerospace actuators, and automated industrial machinery.

For the multi-sinusoidal trajectory, the hybrid controller also demonstrates clear advantages. Although the level of improvement is smaller than that observed for the PTP trajectory, the MSE is reduced from 2.9550 to 1.2566, corresponding to an improvement of 57.48%, while the RMSE decreases from 1.7190 to 1.1210, yielding an improvement of 34.79%. The difference in performance can be attributed to the inherent characteristics of the trajectories. PTP motion involves relatively simple dynamics with distinct start and target points, which allows the Fuzzy-FOPID controller to suppress errors more effectively. In contrast, multi-sinusoidal trajectories introduce rapid variations and overlapping frequency components, thereby increasing the complexity of the control task. Nevertheless, the Fuzzy-FOPID controller still maintains closer alignment with the reference signal compared with the conventional FOPID, demonstrating its ability to deliver consistent accuracy under demanding conditions. The reduction in error values further highlights the controller's capability to manage continuous variations in the input while ensuring accurate trajectory tracking. In practical applications, this translates into smoother actuator operation during dynamic conditions, while reducing the likelihood of overshoot or unwanted oscillations in complex motion profiles.

Overall, the results presented in Table 2 confirm the effectiveness of combining fuzzy logic with fractional-order PID control. The integration significantly reduces tracking errors, enhances accuracy, and ensures reliable performance across different trajectory types. Beyond the numerical improvements, the reduction in absolute error values carries important practical implications: it improves motion quality, decreases mechanical wear, and contributes to more efficient system operation. These findings establish the Fuzzy-FOPID controller as a precise and effective solution for trajectory tracking in industrial hydraulic actuator systems, offering clear advantages over conventional linear controllers.

**Table 2.** Error tracking performance of control design

Tracking Response	Control Strategy	MSE (mm <sup>2</sup> )	Percent of Improvement (%)	RMSE (mm)	Percent of Improvement (%)
Point-to-Point	FOPID	0.8446		0.9190	
	FUZZY-FOPID	0.0239	97.1703	0.1547	83.1665
Multi-sinusoidal	FOPID	2.9550		1.7190	
	FUZZY-FOPID	1.2566	57.4755	1.1210	34.7877

Fig. 14 and Fig. 15 present the tracking error responses for the PTP and multi-sinusoidal trajectories. The results demonstrate that the hybrid Fuzzy-FOPID controller consistently achieves lower error levels compared with the conventional FOPID controller. The PTP trajectory in Fig. 12, the FOPID controller shows large fluctuations, with error values exceeding  $\pm 2$  mm during sudden transitions. In contrast, the Fuzzy-FOPID controller maintains the error within a much narrower range that typically below  $\pm 0.5$  mm. This improvement reflects the contribution of the fractional-order terms which provide accurate baseline control together with the fuzzy component. Its adaptively suppresses error spikes. As a result, the hybrid controller achieves smoother tracking and more accurate positioning. In Fig. 13, both controllers are challenged by rapid changes and overlapping frequency components for multi-sinusoidal trajectory. The FOPID controller exhibits oscillatory errors reaching up to  $\pm 3$  mm, whereas the Fuzzy-FOPID reduces the amplitude by approximately 30–40%. Although there are intervals where both controllers produce similar error levels, the hybrid controller shows

clear advantages in negative displacement regions and during rapid variations. These improvements can be attributed to the adaptive behavior of the fuzzy component, which continuously modifies the control effort to maintain closer alignment with the reference signal.

The figures confirm that the FOPID controller alone can achieve reasonable tracking but struggles with nonlinear and time-varying conditions. The integration of fuzzy logic into the FOPID framework significantly enhances adaptability, yielding lower absolute errors, reduced oscillations, and more reliable tracking. These outcomes emphasize that the fractional-order terms contribute to precision and flexibility, while the fuzzy layer provides the adaptability needed for robust performance under diverse trajectories.

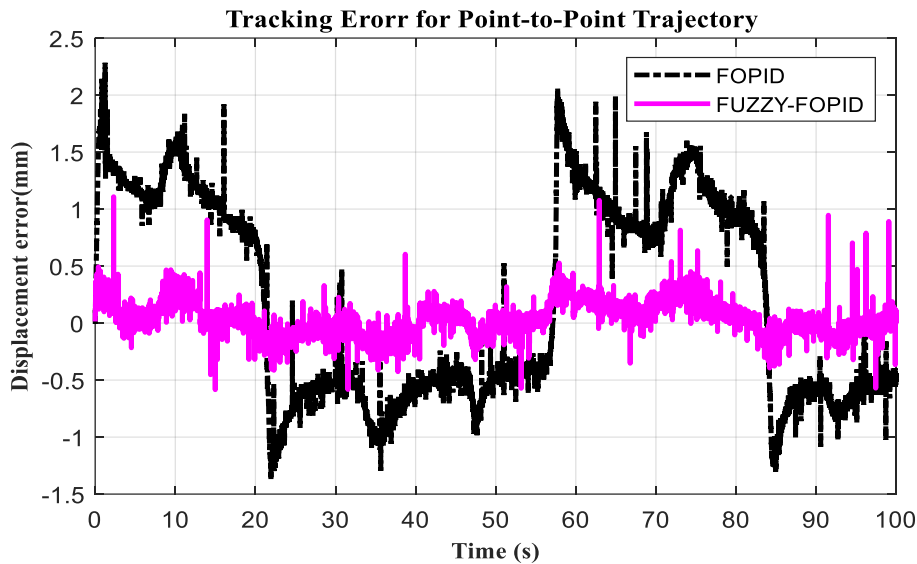


Fig. 14. Tracking error for point-to-point trajectory

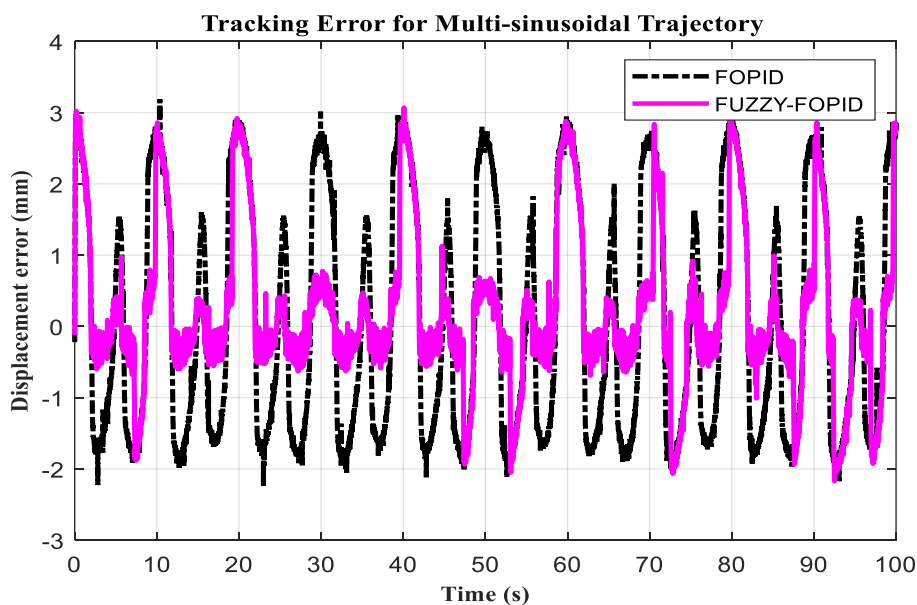


Fig. 15. Tracking error for multi-sinusoidal trajectory

#### 4. Conclusion

In summary, this study has demonstrated that the nonlinear dynamics of the IHA system can be effectively controlled using both FOPID and HF-FOPID controllers. Experimental validation on the hardware platform confirmed that the integration of fuzzy logic with fractional-order control

significantly improves trajectory tracking accuracy and overall system performance. The HF-FOPID controller effectively addresses challenges such as nonlinearities, parameter uncertainties, and dynamic variations that are inherent to hydraulic actuators. Between the two approaches, the HF-FOPID consistently achieved the best results, with the lowest error values across all trajectory tracking tests. For the PTP trajectory, improvements in MSE and RMSE were recorded at 97.17% and 83.17%, respectively, while for the multi-sinusoidal trajectory, the improvements were 57.48% and 34.79%. These outcomes confirm that the hybrid controller provides accurate, stable, and reliable tracking under different motion conditions. Although the proposed controller has shown strong experimental performance, the present study was conducted under fixed operating conditions and specific trajectory profiles. Factors such as varying supply pressures, external load disturbances, and real-time computational constraints were not fully addressed. These aspects represent important directions for future work to further enhance the applicability of the proposed controller in industrial environments.

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